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Loyalty Discounts

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Abstract

This paper considers the use of loyalty inducing discounts in vertical supply chains. An upstream manufacturer and a competitive fringe sell differentiated products to a retailer who has private information about the level of stochastic demand. We provide a comparison of market outcomes when the manufacturer uses two-part tariffs (2PT), all-unit quantity discounts (AU), and market share discounts (MS). We show that retailer's risk attitude affects manufacturer's preferences over these three pricing schemes. When the retailer is risk-neutral, it bears all the risk and all three schemes lead to the same outcome. When the retailer is risk-averse, 2PT performs the worst from manufacturer's perspective but it leads to the highest total surplus. For a wide range of parameter values (but not for all) the manufacturer prefers MS to AU. By limiting the retailer's product substitution possibilities MS makes the demand for manufacturer's product more inelastic. This reduces the amount (share of total profits) the manufacturer needs to leave to the retailer for the latter to participate in the scheme.

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1 Introduction

A loyalty discount is the practice that implicitly or explicitly makes discounts conditional on the share of a buyer's purchases made from a supplier within a given period. The discount is typically applied in a rollback format. Once a buyer qualifies, it receives a discount not only on those purchases above the target, but on all purchases in the period. In most cases, it is difficult to link these discount programs to particular instances of economies of scale. The latter can occur at overall production level or in fulfilling a specific order, but are less likely to relate buyer with private information about uncertainty affect comparisons between these contracts.

In the analysis of vertical chains the tension between efficient surplus extraction and maximization of surplus is thoroughly studied as a principal-agent problem where the retailer has private information related to uncertainty. In contrast to the principle-agent literature where different risk attitudes of the two parties play a central role, previous work that studies motives for various pricing schemes assumes that both upstream and downstream firms are risk-neutral. It is quite plausible that a manufacturer that deals with many retailers in different local markets (potentially subject to uncorrelated shocks) behaves as risk-neutral. But, it is less likely that a retailer would agree to bear all the market risk by signing a contract which aims to induce certain level of purchases at no additional cost to the manufacturer. In effect, our analysis suggests that the differences in attitude towards risk across the vertical chain can explain emergence of different types of loyalty inducing contracts.

In this study, we show that in the presence of uncertainty, if the retailer is infinitely risk averse, the manufacturer strictly prefers market share and all-unit quantity discounts to two-part tariffs. Using a linear demand system, we also show that, for a wide range of parameters, the manufacturer strictly prefers market share discounts to all-unit quantity discounts, and that welfare is highest under two-part tariffs. Private incentives for the use of market-share discounts are driven by their ability to induce the retailer to act on a target share. This reduces the elasticity of retailer's demand for manufacturer's product. However, while a market-share discount limits substitution of the manufacturer's product with the competing product, it still allows the retailer to use private information and respond to actual market conditions which affect both products. Even if implementation of market-share discounts requires costly monitoring of rival sales, there is a non-trivial range of costs for which the supplier might still strictly prefer using a market share discount to using two-part tariffs or all-unit quantity discounts. The importance of the retailer's risk attitude is indicated by the fact that, under risk neutrality, the manufacturer is indifferent between two-part tariffs, market share discounts, and all-unit quantity discounts. In

$q = (q\ ; q\ )$ is the vector of chosen quantities. $P_i(q) \quad C$ and $@P_i=@q_i < 0$ whenever $P_i(q) > 0$ for $i = 1; 2$. The parameter is a discrete random variable which captures potential demand uncertainty common to both products. It takes with probability p a low value $(\ _L)$ and with probability $1 \quad p$ a high value $(\ _H)$. Let $E(\ )$ be the expectation of and $P_i(0;\ _L) > 0$: Shocks in di¤erent downstream markets are assumed to be iid.

A retailer chooses q and q to maximize its pro…ts, $(q;\ ) = R(q;\ ) \quad wq \quad F$; where $R(q;\ ) = P\ (q;\ )q\ + P\ (q;\ )q$ is its revenue. For a given w; $(q) \quad C$ is strictly concave and submodular $(@\ =@q\ @q\ < 0)$. The retailer's outside option consists of selling only the competitively supplied variety. Then, if the retailer rejects the manufacturer's o¤er, it chooses q

2PT only by inducing the retailer to act on the threshold. If a retailer facing an all-unit quantity to act on the threshold both when demand is low and when demand is high, there exist an AU contract that induces the retailer to act on the threshold only when the demand is low which provides higher expected pro…ts to the manufacturer.

Let us now consider market share discount contacts. With this type of contract, the retailer quali…es for a price discount if at least a percentage of its purchases are made from the manufacturer. If such a contract induces the retailer to act on the share target, it limits the retailer's substitution of the manufacturer's product with the competitively supplied alternative in response to an increase in price. This reduces the market for substitutes of manufacturer's product and allows it to charge a higher unit price as elasticity of the retailer's demand falls.

A retailer that acts exactly on the share threshold chooses q and q to maximize:

$$P\ (q;\ )q\ + P\ (q;\ )q \quad wq \quad F;$$

$$\text{subject to } q\ =\ (q\ + q\ ):$$

Let s = =(1) (note that (0; 1) s <), then the constraint requires that q = sq : Substituting the constraint, it follows that for a retailer that acts exactly on the threshold, the quantity of good 2, q**(w; s;); maximizes:

$$_{MS} = P\ (sq\ ; q\ ;\ )sq\ + P\ (sq\ ; q\ ;\ )q \quad wsq \quad F:$$

The …rst order condition of the maximization problem is:

$$\frac{@P}{@q} s\ q^{**} + \frac{@P}{@q} sq^{**}$$

Then, there exists > 0 such that $U_{MS}(w^*_{PT} + \;; s_{PT}) > U^*_{PT}$. ■

The proof of Proposition 3 shows that there exists a MS contract that results in higher upstream pro…ts than the optimal 2PT. The proof uses an MS contract which induces the retailer to act on the threshold only when the demand is low. However, this is not necessarily true for the optimal MS contract. In the linear demand example presented below, the optimal MS contract actually induces both types to act on the share threshold. Notice that a retailer acting on a share target still makes use of its private information on demand. By correctly adjusting its purchases of the competitively supplied product it can meet the threshold while allowing its choices to respond to uncertainty.

The manufacturer prefers MS contracts to 2PT because setting a share threshold limits product substitution and makes retailer's demand for its product more inelastic. Lower price-elasticity allows the manufacturer to charge a higher unit-price and transfer more surplus upstream, so that retailer's participation constraint is met at a lower cost.

A similar e¤ect is in operation under an AU contract when the demandainn

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$$q^* = \frac{(1 \quad)(a + \quad) \quad w}{2(1 \quad)} \text{ and } q^* = \frac{(1 \quad)(a + \quad) + \quad w}{2(1 \quad)}; \tag{7}$$

$$^* = \frac{2(1 \quad)a(a \quad w) + w}{4(1 \quad)} + \frac{2a \quad w}{2(1 + \quad)} + \frac{1}{2(1 + \quad)} \quad F;$$

where $\quad _L; \ _H$: The retailer's outside option is $R^*_O(\ ) = (a + \ ) = 4$ and its participation constraint in this case requires that $^*(\ _L) \quad R^*_O(\ _L)$.

Manufacturer's equilibrium choices and profits are, respectively:

$$w^*_{PT} = (1 \quad)(1 \quad p)(\ _H \quad _L),\ F^*_{PT} = \frac{(1 \quad)[a \quad (1 \quad p)(\ _H \quad _L) + \ _L)]}{4(1 + \quad)}; \text{ and}$$

$$U^*_{PT} = \frac{(1 \quad)[(a + \ _L) \ + (1 \quad)(1 \quad p)\ (\ _H \quad _L)\ ]}{4(1 + \quad)}:$$

In the Appendix we summarize the equilibrium outcomes when the retailer is infinitely risk averse in the linear demand example.

Consider now an all-unit quantity discount contract. It can be shown that the optimal AU contract induces only the low type to act on threshold, while the high type purchases above the threshold. The optimal second stage revenue of the low type retailer is

$$\hat{R}(q^T;\ _L) = \frac{(a + \ _L)}{4} \quad (1 \quad)(q^T) \ + (1 \quad)(a + \ _L)q^T.$$

The participation constraint is binding and, in equilibrium,

$$q^T = \frac{(a + \ _L)(1 + p) \quad (1 \quad p)(\ _H \quad _L)}{2(1 + \quad)(1 + p)};\ w^*_{AU} = \frac{(1 \quad)(\ _H \quad _L)}{1 + p} \text{ and}$$

$$F^*_{AU} = \frac{(1 \quad)[a \quad (\ _H \quad 2\ _L)]}{4(1 + \quad)} + \frac{p(1 \quad)(\ _H \quad _L)[a \quad (\ _H \quad 2\ _L)]}{2(1 + \quad)(1 + p)}:$$

As in the case of a 2PT contract, risk aversion and the related insurance provision to the retailer raise the wholesale price above the level that maximizes total surplus. However, although

There are two underlying effects behind this welfare comparison. First, when demand is low the distortion in the retailer's sales due to double marginalization is lower under the optimal AU contract because sales of the manufacturer's good are determined by the threshold rather than the first order condition.[11] Second, the higher wholesale price increases the negative welfare impact of double marginalization when demand is high and the retailer's sales are governed by the first order condition.[12] The second effect is stronger le(s)-8-e(s)-8-e(sog)-1(d)45-1(d)(sog)-1(d)45-15g(s)-

Proposition 4 Under demand uncertainty, with an in…nitely risk-averse retailer and linear demand, expected total welfare and consumer surplus are highest under 2PT contract. Expected consumer surplus is lowest under MS contract. For $p < (1 + \ )=2$; from both private and social viewpoints, MS contract outperforms AU contract.

The intuition underlying Proposition 4 is related to the delegation problem (see, for instance, Rey and Tirole (1986)). Under uncertainty the manufacturer pursues to exploit market power in the vertical chain and to o¤er insurance to the risk-averse retailer. In our model, the fact that the retailer is a multiproduct …rm a¤ects both upstream objectives. An integrated monopolist can exploit market power optimally in the vertical structure. It passes the product downstream at marginal cost and, under uncertainty, it chooses $q^{VI} = q^{VI} = (a + \ )=[2(1 + \ )]$: The retail quantity responds to the uncertainty, and the share of manufacturer's product is constant across states. The vertically integrated share of manufacturer's product is $^{VI} = 50\%$ $(= q^{VI}=(q^{VI} + q^{VI}))$:[15]

When dealing with a risk averse retailer, the manufacturer cannot extract the incremental surplus from the retailer through the franchise fee as the retailer requires insurance from market risk, and is forced to sell its product above marginal cost. With a 2PT contract, the retailer chooses quantities $q^{PT} < q^{VI}$ and $q^{PT} > q^{VI}$ that respond to the uncertainty.[16] Due to the higher unit price, the share of manufacturer's product is lower, $^{PT}(\ ) = q^{PT}=(q^{PT} + q^{PT}) < 50\%$ (and varies across states, $^{PT}(_L) < \ ^{PT}(_H)$), as the retailer purchases more of the substitute product. Under the low demand, the AU contract induces the retailer to act on threshold. This limits retailer's ability to cut down the share of manufacturer's product when facing low demand ($^{AU}(_L) = q^T$

to bear all market risk. So, in this case there is no con‡ict between surplus extraction incentives and insurance provision. Then, the manufacturer passes the product to the retailer at marginal cost ($w_{RN} = 0$) and appropriates surplus through the franchise fee that is equal to retailer's expected pro…t net of its outside option ($F_{RN} = E(R^*(0;)) \quad E(R_O^*( )))$. Two-part tari¤s, all-unit quantity discounts, and rollback market share discounts are all equally e¤ective tools to maximize and extract surplus in the vertical chain.

Proposition 5

Amongst possible extensions are generalizations in three directions. The two products sold by the retailer may eventually be vertically di¤erentiated. It is interesting to see if the results extend to more general downward sloping demand functions, or to more general utility functions of the risk averse retailer. So far, we concentrated on non-contingent contracts (used in many identical and independent retail markets where the manufacturer operates). However, such contract comparison might shed light on the use of loyalty rebates also when the contracts are contingent on demand realizations.

5 Appendix

5.1 Risk-Neutral Retailer

Proof of Proposition 5. With uncertain demand, the risk-neutral retailer makes quantity choices after observing the realized demand. Hence, the second stage optimizations presented in subsection 3.1 still apply. But, since contracts are agreed upon before the resolution of uncertainty, a di¤erent risk attitude changes the …rst stage optimization. When the manufacturer faces a risk neutral retailer, the participation constraint requires retailer's expected pro…t to be at least equal to retailer's expected outside option.

Under a 2PT contract, the upstream manufacturer chooses w and F to maximize

$$pwq^*(w;\ _L) + (1 \quad p)wq^*(w;\ _H) + F \quad \text{subject to}$$

$$p(R^*(w;\ _L) \quad wq^*(w;\ _L)) + (1 \quad p)(R^*(w;\ _H) \quad wq^*(w;\ _H)) \quad F \quad pR^*_O(\ _L) + (1 \quad p)R^*_O(\ _H):$$

The constraint is increasing in the franchise fee so the supplier chooses the unit price to maximize $pR^*(w;\ _L) + (1 \quad p)R^*(w;\ _H)$: It follows that the optimal unit price w_{RN} satis…es the …rst order condition

$$p(\frac{@R(\ _L)}{}$$

order condition $(1 \quad p)(\frac{@R \quad _H}{@q_1}\frac{@q_1^* \quad _H}{@w} + \frac{@R \quad _H}{@q_2}\frac{@q_2^* \quad _H}{@w}) = 0$: By a similar argument as in the case of a 2PT, it follows that the optimal unit price and franchise fee are given by (9). In addition, $q^T = \arg\max p\hat{R}(q^T; \ _L) = q^*(0; \ _L)$: Clearly the manufacturer cannot improve upon this contract. The optimal 2PT and AU contracts result in the same output levels. ■

Finally, consider a MS contract that induces the retailer to act on the threshold always. Then, the supplier chooses w; s and F to maximize

$$pwsq^{**}(w; s; \ _L) + (1 \quad p)wsq^{**}(w; s; \ _H) + F \quad \text{subject to}$$
$$p(R^{**}(w; s; \ _L) \quad wsq^{**}(w; s; \ _L)) + (1 \quad p)(R^{**}(w; s; \ _H) \quad wsq^{**}(w; s; \ _H)) \quad F$$
$$pR_O^*(\ _L) + (1 \quad p)R_O^*(\ _H):$$

The constraint is increasing in the franchise fee, so the supplier chooses w and s to maximize $pR^{**}(w; s; \ _L) + (1 \quad p)R^*(w; s; \ _H)$: Then, the unit price satis…es

$$p(\frac{@R}{@q}s\frac{@q^{**}(\ _L)}{@w} + \frac{@R}{@q}\frac{@q^{**}(\ _L)}{@w}) + (1 \quad p)(\frac{@R}{@q}s\frac{@q^{**}(\ _H)}{@w} + \frac{@R}{@q}\frac{@q^{**}(\ _H)}{@w}) = 0: \qquad (10)$$

From (6), using env

Table2: Uncertainty and In…nite Risk Aversion

Contract	2PT	AU	MS
q	$\underline{a \quad - \quad -p \quad {}_{H} - {}_{L}}$	$q^L = \frac{p \; a \quad {}_{H} - \quad {}_{H} - {}_{L}}{p}$ $q^H = \frac{p \; a \quad {}_{H} - \quad {}_{H} - {}_{L}}{p}$	$\underline{a \quad - \quad -p \quad {}_{H} - {}_{L}}$
q	$\underline{a \qquad -p \quad {}_{H} - {}_{L}}$	$q^L = \frac{p \; a \quad {}_{L} \qquad -p \quad {}_{H} - {}_{L}}{p}$ $q^H = \frac{p \; a \quad {}_{H} \qquad {}_{H} - {}_{L}}{p}$	$\underline{a \quad - \quad -p \quad {}_{H} - {}_{L}}$
P	$\underline{a \quad - \quad -p \quad {}_{H} - {}_{L}}$	$p^L = \frac{p \; a \quad {}_{L} \quad - \quad -p \quad {}_{H} - {}_{L}}{p}$ $p^H = \frac{p \; a \quad {}_{H} \quad - \qquad {}_{H} - {}_{L}}{p}$	$\underline{a \qquad -p \quad {}_{H} - {}_{L}}$
P	$\underline{a \quad}$	$\underline{a \quad}$	$\underline{a \qquad -p \quad {}_{H} - {}_{L}}$
R_L	$\underline{a \quad {}_{L}{}^{2}}$	$\underline{a \quad {}_{L}{}^{2}}$	$\underline{a \quad {}_{L}{}^{2}}$
R_H	$\underline{- \quad {}_{H} - {}_{L} \quad a \; E \quad - \quad {}_{H} - {}_{L}} + \underline{a \quad {}_{H}{}^{2}}$	$\frac{- \quad {}_{H} - {}_{L} \quad a \quad {}_{H} \quad p^2 - \quad p \quad {}_{H} - {}_{L}}{p^2} + \underline{a \quad {}_{H}{}^{2}}$	$\underline{{}_{H} - {}_{L} \quad - \quad a \quad {}_{H} \quad {}_{L} - \quad -p \quad {}_{H} - {}_{L}} + \underline{a \quad {}_{H}{}^{2}}$
w	$(1 \quad)(1 \quad p)(\; {}_{H} \quad {}_{L})$	$\frac{- \quad {}_{H} - {}_{L}}{p}$	$2(1 \quad p)(\; {}_{H} \quad {}_{L})$
F	$- \quad a \quad {}_{L} - \quad -p \quad {}_{H}(1 -$		

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