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Price Competition with Consumer Confusion

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1 Introduction

Sellers often use various ways to convey price information to consumers. Retailers use different discount methods to promote their products, such as direct price reductions, percentage discounts, volume discounts, or vouchers.¹ Some restaurants, hotels, and online booksellers offer a single price, while others divide the price by quoting table service, breakfast, internet access, parking, or shipping fees separately. Airlines and travel agencies charge card payment fees in different ways. For instance, Wizz charges a flat £4 per person, while Virgin Atlantic charges 1.3% of the total booking.² Retailers offer store cards with diverse terms such as “10% off first shop if opened online or 10% off for the first week if opened in store”, “500 bonus points on first order”, or “£5 voucher after first purchase”. Financial product prices are also often framed distinctively: mortgage arrangement fees might be rolled in the interest rate or not; some loans may specify the monthly interest rate, while others the annual interest rate. In some cases (e.g., supermarket promotions), sellers also change their price presentation formats over time.

firms adopt mixed strategies that randomize on both price frames and prices, and make strictly positive profits in an otherwise homogeneous product market. Moreover, as the number of firms increases, it becomes more difficult to obfuscate price comparisons by adopting different frames, and firms use complex price frames more often. As a result, more competition might actually boost profits and harm consumers. Our model suggests that in the presence of price framing, a standard competition policy approach may have undesired effects on consumer welfare.

Marketing research provides evidence that consumers have difficulties in comparing prices that are presented differently or prices that are complicated (see, e.g., Estelami, 1997, Morwitz et al., 1998, and Thomas and Morwitz, 2009). Economics experiments (see, e.g., Kalayci and Potters, 2011, and Kalayci, 2011) show that increasing the number of product attributes or price scheme dimensions can create confusion and lead to suboptimal consumer choices. We explore two sources of consumer confusion due to price presentation: frame differentiation (when firms adopt different frames) and frame complexity (when firms use a common but complex frame).

Consider, for instance, the following two frames: “price per unit” and “price per kilogram”.⁵ In this case, comparing two prices in the same frame is easy, but comparing a price per unit with a price per kilogram might be difficult for some consumers. Here, frame differentiation is the confusion source. Other examples of incompatible price formats are price incl. VAT vs. price excl. VAT, flat card payment fees vs. percentage ones, and monthly interest rate vs. annual interest rate quotations on loans.

Now consider the frames “price incl. shipping fee” and “price plus shipping fee”. Ranking all-inclusive prices is easy and, as before, comparing prices in different frames might be difficult. However, in this case comparing prices that quote separately the shipping fees may also be confusing if the fees vary across sellers. Here, frame complexity arising from the use of involved formats (two-dimensional prices) is also a source of consumer confusion. This is true in other settings (e.g., in financial services or utility markets) where some frames are involved multi-part tariffs.⁶ For instance, mortgage deals with the service fee quoted separately are usually harder to compare than deals with the service fees rolled in the interest rate. When both sources of confusion coexist, it is not obvious which of them is more likely to confuse consumers. The answer depends on the microfoundations of confusion, which will be discussed in the modelling

which generates both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random). As a result, firms will also randomize on prices in equilibrium. This prediction is consistent with casual observations in many markets. Grocery stores and online

evidence on obfuscation strategies in online markets where retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. Carlin (2009) and Ellison and Wolitzky (2008) address this issue in the information search framework where each firm chooses both a price and a price complexity level. They argue that if it is more costly for consumers to assess complex prices, each firm will individually increase price complexity to reduce consumers' incentives to gather information and weaken price competition.⁹ Our model also considers price complexity, but it incorporates the effect of price frame differentiation and regards it as an important source of market complexity. In particular, in our model whether a firm's frame choice can soften price competition also depends on rivals' frame choices. This strategic dependence induces firms to randomize on frames. So our model predicts that firms tend to adopt different price frames or change their price frames over time.¹⁰

In a closely related paper, Piccione and Spiegler (2012) also examine frame-price competition. The

boundedly rational consumers to compare framed prices leads to equilibrium frame dispersion. Our study is also related to the literature on consumer search and price dispersion. But, we focus on how firms may confuse consumers by mixing their frame choices, and in our model price dispersion is a by-product of frame dispersion.

2 The Duopoly Model

of confused consumers for all the frame profiles, where z_i is the frame chosen by firm i and z_j is the frame chosen by firm j .

Table 1: Confused consumers

$z_i \setminus z_j$	A	B
A	0	1
B	1	2

We assume that nobody is confused if both firms use A for expositional reasons. The main results hold qualitatively if a fraction of consumers also get confused in this case, provided that $\alpha \leq \beta$ and $\alpha > \beta$.

Then, firm i 's profit is

$$\pi_i(p_i; p_j; z_i; z_j) = p_i \cdot \alpha_{z_i; z_j} + q_i(p_i; p_j) \cdot \beta_{z_i; z_j};$$

where $\alpha_{z_i; z_j}$ is presented in Table 1 and $q_i(p_i; p_j)$ is given by (1).

In our model, confused consumers do not pay more than their reservation price equal to v . Arguably, if price framing prevents a consumer from comparing competing offers, it may also prevent her from accurately comparing framed prices and her willingness to pay. In this case, one way to justify our assumption is that consumers can figure out at checkout (or after purchase) if a product's price exceeds their valuation and can decline to buy it (or return it). Given such ex-post participation constraint, firms have no incentive to charge prices above v .¹¹ In addition, confused consumers are assumed to be unable to understand the relationship between price frames and prices. For example, even if a particular frame is always associated with higher prices, confused consumers are unable to infer prices from the price frame. This may be the case if consumers who lack the ability to compare prices are also unable to understand the market equilibrium. We revisit this issue in Section 4.

Our model explores two sources of consumer confusion: frame differentiation (i.e., prices are presented in incompatible formats) and frame complexity (i.e., prices are presented in a common involved format). If $\beta = 0$ (i.e., if frame B is also a simple frame), frame differentiation is the sole source of consumer confusion and it is captured by α . If $\beta > 0$, frame complexity is also a source of consumer confusion. When consumers face the frame profile $(B; B)$, a

of α_1 and α_2 reflects the relative importance of frame differentiation and frame complexity as sources of consumer confusion.

The relative role of the two confusion sources and their relevance in the marketplace stem from the microfoundations of consumer confusion. We present below two possible interpretations.

Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$). When consumers face a simple frame A and a complex frame B, to compare the two offers they need to convert the price in frame B into a single all-inclusive price. Imagine that due to differences in numeracy skills, some consumers are able to make a correct conversion, while others are not. We assume that those who are unable to convert get confused and end up choosing randomly. When consumers face two offers in frame B, those who are able to convert B into A should still be able to compare. Moreover, those with poor numeracy skills may now benefit from format similarity. For example, if frame B is a two dimensional price and one offer dominates the other in both dimensions, then even those who are unable to convert will make the right choice. That is, similarity between the price formats may mitigate the confusion caused by frame complexity.¹² This is obvious, for example, when B is “price plus VAT” and the same tax rate applies. In this example, frame similarity rules out confusion (and B can be regarded as a simple frame).

Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$). Consumers might be able to convert a price presented in frame B into a simple price in frame A, but this requires costly information processing and consumers may decide whether or not to make the conversion. When they give up making the conversion, they end up confused. If confusion stems from this conversion cost, a consumer is more likely to give up the effort when she compares two complex prices than when she compares one complex price with a simple one. Then, the frame profile $B; B$ leads to more confused consumers than the profile $A; B$.

We use a reduced-form approach and do not explicitly model the comparison procedures that may lead to confusion. In reality, there may be several confusion mechanisms so that both cases of $\alpha_1 > \alpha_2$ and $\alpha_2 > \alpha_1$ are worth exploring.

Finally, in our setting confused consumers’ choices are assumed to be totally independent of firms’ prices. This is a tractable way to capture the idea that confusion in price comparisons reduces consumers’ price sensitivity and weakens price competition. An alternative (but less tractable) model might assume that price framing leads to noisy price comparisons. Suppose firm i charges a price p_i . If it uses the simple frame A, consumers will understand its price perfectly. In contrast, if it uses frame B, consumers will perceive its price as $p_i + \epsilon_i$, where ϵ_i is a random variable that captures possible misperceptions. Then, for example, if firm i adopts

¹²Even if there is no clear dominance relationship between offers, frame similarity may still facilitate comparison of prices framed in B. Take for example two offers in frame B: (1) £ p_1 plus £ s_1 shipping, and (2) £ p_2 plus £ s_2 shipping. When a consumer compares them, she may assess different components separately. The base price in (2) is about p higher than in (1), but the shipping fee in (2) is about p cheaper than in (1), so (2) is a better deal than (1). However, if the consumer needs to compare, say, (1) with a single price £ $p_1 + s_1$, it seems plausible that she has to convert (1) into an all-inclusive price first, which is more demanding in calculation and so it may block the comparison.

the relatively complex frame B and firm j adopts frame A, consumers perceive their prices as p_i and p_j , respectively. As a result, demand becomes less elastic compared to the case where both firms use frame A. (This is reflected by $\epsilon_1 >$

profits as some consumers are confused by “frame differentiation” and shop at random. For $\alpha_2 > \alpha_1$, Lemma 1 also shows that in equilibrium, the firms cannot rely on only one confusion source. Otherwise, a firm using frame B has a unilateral incentive to deviate to the simpler frame A to attract price aware consumers. But, if $\alpha_1 > \alpha_2$, there is an equilibrium with both firms using frame B, as a unilateral deviation to frame A does not change the composition of consumers in the market.

With probability α , the rival uses A so that a fraction α_1 of the consumers are confused (by frame differentiation) and shop randomly. With probability $1 - \alpha$, the rival also uses B so that a fraction α_2 of the consumers are confused (by frame complexity) and shop randomly.¹⁶

The nature of the equilibrium depends on which confusion source dominates. Intuitively, when $\alpha_1 < \alpha_2$, if a firm shifts from frame A to B, more consumers get confused regardless of its rival's frame choice. Thus, each firm charges higher prices when it uses frame B than when it uses frame A. For $\alpha_1 > \alpha_2$, when a firm shifts from frame A to B, more consumers get confused if its rival uses A, while fewer consumers get confused if its rival uses B. Hence, there is no obvious monotonic relationship between the prices associated with A and B. Below we analyze these two cases separately.

- Frame differentiation dominates frame complexity: $\alpha_1 \leq \alpha_2 < \alpha$

The unique symmetric equilibrium in this case dictates $F_A(p) = F_B(p)$ and $S_A = S_B = p_0$; (see Appendix A for the proof). That is, a firm's price is independent of its frame. Let $F(p)$ be the common price cdf and $x(p) \equiv 1 - F(p)$. Then, using the profit functions (2) and (3) and the frame indifference condition $\pi_A(p) = \pi_B(p)$, we obtain

$$1 - \frac{1}{\alpha}$$

Then the boundary price p_0 is defined by $x(p_0)$ and one can check that $p_0 \in [p_1; p_2]$. The price cdf for a higher α_1 (α_2) first-order stochastically dominates that for a lower α_1 (α_2). This is consistent with the observation that confusion benefits firms and harms consumers. We summarize these findings below:

Finally, $F_z(p)$ is determined by $z; p$. Explicitly, we have

$$x_A(p) - z = 1 - \frac{z}{p} \quad (9)$$

and

$$1 - z = 2 - 2x_B(p) - \frac{z}{p} \quad (10)$$

The boundary prices p_0^A and p are defined by $x_A(p_0^A) = z$ and $x_A(p) = 1 - z$, respectively. Both of them are well defined with $p_0^A < p$. We summarize these results below:

Proposition 2 In the duopoly model,

(i) if $\alpha_1 < \alpha_2 <$

pricing stage echoes part (b) in the proof of Lemma 1, and each firm makes $\pi_2 = \pi_1$ (which is greater than π_1). In sum, in a two-stage game, a pure-strategy equilibrium is more likely and firms tend to refrain from mixing on frames. But, there is still consumer confusion in the market either because firms adopt different frames or because they use complex frames.

3 The Oligopoly Model

In this section, we develop a general oligopoly version of the model to analyze the impact of competition on market outcomes in the presence of price framing.

Consider a homogeneous product market with $n \geq 2$ identical sellers and, as before, two categories of frames, A and B. A is a simple frame so that all prices in this frame are comparable. B is potentially complex so that with probability $\alpha \geq 0$ the consumers cannot compare prices in this frame. Consumers can also be confused by frame differentiation and so unable to compare prices in different frames with probability $\beta > 0$. In continuation, we focus on the case where confusion due to frame differentiation is independent of confusion due to frame complexity. However, depending on the microfoundations, the two types of confusion may be correlated. We argue in Section 4 that our analysis and its main insights carry over to the case where the two confusion sources are dependent.

In duopoly, $\pi_1 = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \alpha \right) = \frac{1}{2} (1 + \alpha)$. In oligopoly, $\pi_1 = \frac{1}{n} \left(\frac{1}{2} + \frac{1}{2} \alpha \right) = \frac{1}{n} (1 + \alpha)$.

Example 2 Consider a case with n firms. Firm 1 uses frame A and charges price p_1 , and firms 2 and 3 use frame B and charge p_2 and p_3 , respectively. If $\alpha_1 > \alpha_2$ and $\alpha_2 > \alpha_3$ (i.e., frame B is also simple), then only frame differentiation causes confusion. All consumers can accurately compare p_2 with p_3 since they are presented in the same frame, but cannot compare p_1 with either p_2 or p_3 . So consumers are neither fully aware nor totally confused.

So, a major question is how does a consumer choose from a “partially ordered” set in which some pairs of alternatives are comparable, but others are not. Note that this is not an issue in the duopoly model. To address this consumer choice issue, following the literature on incomplete preferences, we adopt a dominance-based consumer choice rule. The basic idea is that consumers only choose, according to some stochastic rule, from the “maximal” alternatives which are not dominated by any other comparable alternative. From now on, we use “dominated” in the following sense.

Definition 1 For a consumer, firm i 's offer $(z_i; p_i) \in \{A; B\} \times \mathbb{R}^+$ is dominated if there exists firm $j \neq i$ which offers alternative $(z_j; p_j) < (z_i; p_i)$ and the two offers are comparable.

Notice that for any consumer, the set of maximal or undominated alternatives is well-defined and comparable.

1. Consumers first eliminate all dominated offers in the market.
2. They then buy from the undominated firms according to the following stochastic purchase rule (which is independent of prices): (i) if all these firms use the same

3.1 Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$)

We analyze now the case where consumers are more likely to be confused by frame differentiation than by the complexity of frame B (that is, $\alpha_1 > \alpha_2$). For simplicity, we first focus on the polar case in which prices in different frames are always incomparable (i.e., $\alpha_1 = 0$). We then discuss how the main results can be extended to the case with $\alpha_1 < \alpha_2$. All proofs missing from the text are relegated to Appendix B.1.

Lemma 4 in Appendix B.1 shows that there is no pure-strategy equilibrium when $\alpha_2 > \alpha_1$. If $\alpha_2 = \alpha_1$ (both frames are simple) and $n \geq 2$, there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms price at marginal cost. However, for any $n \geq 2$, there is a symmetric mixed-strategy equilibrium in which firms make positive profits.

A symmetric mixed-strategy equilibrium. Let $(\alpha; F_A; F_B)$ be a symmetric mixed-strategy equilibrium, where α is the probability of using frame A and F_z is a price cdf associated with frame $z \in \{A; B\}$. Let $[p_0^z; p_1^z]$ be the support of F_z . As in Lemma 3, it is clear that F_z is atomless everywhere (as now $\alpha_2 < \alpha_1$). For the rest of the paper,

$$P_{n-1}^k \equiv C_{n-1}^k = \binom{n-1}{k} \alpha^k (1-\alpha)^{n-1-k}$$

denotes the probability that k firms among $n-1$ ones adopt frame A at equilibrium, where C_{n-1}^k stands for combinations of $n-1$ taken k . Recall that $x_z(p) = F_z(p)$.

Along the equilibrium path, if firm i uses frame A and charges price p , its profit is:

$$\pi_i(A; p) = p^{n-1} x_A(p) - p^{n-1} \sum_{k=0}^{n-1} P_{n-1}^k x_A(p^k) - \frac{1}{2} (p - p_0^A)^2 \quad (11)$$

If k other firms also use frame A, firm i has a positive demand only if all other A firms price higher than p . This happens with probability $x_A(p^k)$. Conditional on that, if there are no B firms in the market (if $k = n-1$), then firm i serves the whole market. The first term in $\pi_i(A; p)$ follows from

are not confused buy from firm i only if it offers the lowest price. When $k \geq 1$, firms use frame A (note that only one of them will be undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame B), all B firms are undominated and have demand $\frac{1}{n-k}$ in total. Firm i shares equally this residual demand with the other B firms. If the consumer is not confused by frame complexity, to face a positive demand, firm i must charge the lowest price in group B (this happens with probability $x_B p^{n-k-1}$), in which case it gets the residual demand $\frac{1}{n-k}$.

Note that for $k=1$ price competition can only take place among firms that use the same frame, and so $x_A p$ does not appear in $B; p$ and $x_B p$ does not appear in $A; p$. This also implies that both profit functions are valid even if firm i charges an off-equilibrium price. Thus, the upper bounds of the price cdf's are frame-independent: $p_1^A = p_1^B$. Otherwise any price greater than p_1^z would lead to a higher profit. Then the frame-indifference condition $A; p = B; p$, pins down a unique well-defined $p \in [z; p]$. (See equation (17) in Appendix B.1). Each firm's equilibrium profit is

$$A; p = \frac{1}{n-k} \left(\frac{1}{n-k} - \frac{1}{n} \right) \frac{1}{p} \quad (13)$$

The price distributions F_A and F_B are implicitly determined by $p \in [z; p]$ since any price in the support of

- (i) when n increases from n_1 to n_2 , both α and industry profit π decrease;
- (ii) for any $n \geq n_2$, there exists $\alpha \in (0, 1)$ such that for $n_2 > n_1$, α decreases but industry profit π increases from n_1 to n_2 .

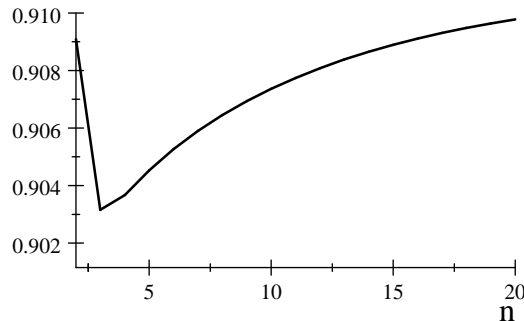


Figure 2: Industry profit and n when $\alpha_1 < \alpha_2$:

Beyond the limit results, numerical simulations suggest that α tends to decrease in n , and industry profit can increase in n for a relatively large α_2 .²² Figure 2 shows how industry profit varies with n when $\alpha_2 > \alpha_1$:

The case with $\alpha_2 < \alpha_1 < \frac{1}{2}$. Price competition can also take place between firms using different frames. Then both $x_A(p)$ and $x_B(p)$ appear in the profit functions $\pi(z; p)$. The more involved related analysis is presented in the supplementary document. There we show that if a symmetric mixed-strategy equilibrium exists, then it still satisfies $p_1^A = p_1^B$. Numerical simulations suggest that greater competition can still have undesired effects (for example, when α_1 is large and α_2 is close to α_1). For example, when $\alpha_1 = 0.8$ and $\alpha_2 = 0.7$, industry profit varies with n in a way similar to Figure 2.

3.2 Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$)

Consider the case where consumers are more likely to be confused by the complexity of frame B than by frame differentiation (i.e., $\alpha_2 > \alpha_1$). Again, we first analyze the polar case in which prices in frame B are always incomparable (i.e., $\alpha_2 = 0$). We then discuss the robustness of our main results to the case with $\alpha_2 < \frac{1}{2}$. The analysis resembles the previous one, so we only report the main results here and relegate the details to Appendix B.2.

Proposition 6 For $n \geq 2$ and $\alpha_2 < \alpha_1 < \frac{1}{2}$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability α and frame B with probability $1 - \alpha$. When a firm uses frame A, it chooses its price randomly according to a cdf F_A defined on $[p_0^A, \infty)$; when it uses frame B, it charges a deterministic price $p = p_0^B$.

²²For a sufficiently small α_2 , increasing the number of firms will lower industry profit. This can be seen when $\alpha_2 = 0$, as $\pi = \alpha n$ (for any n) and industry profit is $n\alpha$, which decreases in n .

Using the equilibrium in proposition 6, we analyze the impact of greater competition on the market outcome. When there are many sellers in the market, the same results as in Proposition 4 for $\alpha_2 > \alpha_1$ hold. That is, $\pi_n < \pi_{n-1}$ and $\pi_{n-1} > \pi_n$. The same intuition applies: in a sufficiently competitive market, the ability of frame differentiation to soften price competition is negligible, and so firms resort to the complexity of frame B.

The following result shows that in the current case greater competition can also improve industry profit and decrease consumer surplus. In particular, this must happen when α_1 is small. The reason is that, for a small α_1 , the complexity of frame B is more effective in reducing price competition, which makes the frequency of using frame B increase fast enough with the number of firms. The resulting market complexity could then dominate the usual competitive effect of larger n . Figure 3 below illustrates how industry profit varies with n when $\alpha_1 < \alpha_2$.²³

Proposition 7 In the case with $\alpha_1 < \alpha_2 < 1$, for any $n \geq 2$, there exists $\alpha_1 \in (0, 1)$ such that for $\alpha_1 < \alpha_1^*$, π_n decreases while industry profit Π_n increases from n to $n+1$.

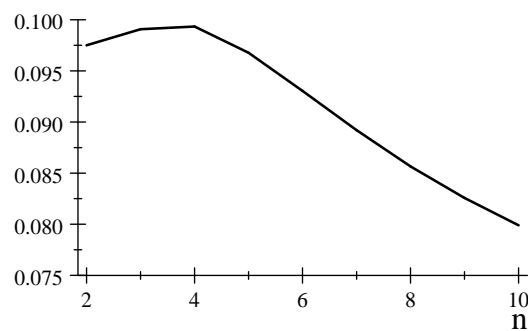


Figure 3: Industry profit and n when $\alpha_1 < \alpha_2$

The case with $\alpha_1 < \alpha_2 < 1$. This analysis is more involved, and we relegate it to the supplementary document. Note that a symmetric separating equilibrium with $S_A = p_0^A; p$ and $S_B = p; p_1^B$, resembling the one in Proposition 6, still exists under some parameter restrictions (when α_1 is not too close to $\alpha_2 < 1$). Also, for fixed $\alpha_2 < 1$, if α_1 is sufficiently small, greater competition can still increase industry profit and harm consumers.

4 Discussion

Comparison with the default-bias choice rule in Piccione and Spiegler (2012). The dominance-based choice rule embeds a simultaneous assessment of competing offers, and a consumer's final choice is not affected by the sequence of pairwise comparisons. This "simultaneous search" feature is suitable in markets where the consumers are not influenced by past experiences (or are

newcomers). Piccione and Spiegler (2012) consider a default-bias model where consumers are initially randomly attached to one brand (the default option), and they shift to another brand only if it is comparable to and better than their default. There, with sequential comparisons, a consumer's final choice depends on her default option.

In duopoly, the default-bias model is equivalent to the simultaneous assessment one (with the random purchase rule for confused consumers).²⁴ This is because, if the two firms' offers are comparable, in both models the better one attracts all consumers, whereas if they are incomparable, in both models the firms share the market equally. But, with more than two firms, the two models diverge. In this case, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

Example 4 There are three firms in the market. Let ω_2 and ω_1 (the only confusion source is frame complexity). Firm 1 adopts frame A and prices at p_1 , while firms 2 and 3 adopt frame B and price at p_2 and p_3 , respectively, with $p_2 < p_1 < p_3$.

The dominance-based rule implies that consumers purchase only from firm 2 since firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2. Now consider the default-bias model. A consumer initially attached to firm 2 does not switch. If she is initially attached to firm 1, she switches to firm 2. However, if she is initially attached to firm 3, she switches to firm 1, but whether she further switches to firm 2 depends on what the choice rule of the default-biased consumer dictates. The rule should specify if the consumer assesses firm 2's offer using her default option (i.e., firm 3) or using her new choice (i.e., firm 2). By contrast, the dominance-based rule applies regardless of the number of firms in the market.²⁵

...rms still randomize on both frames and prices (see the supplementary document for details). However, it is not possible to fully characterize the equilibrium.

Both cases $\alpha_1 > \alpha_2$ and $\alpha_1 < \alpha_2$ can be justified in this setting with noisy price comparisons. To illustrate, suppose ϵ_i is a random variable with the standard normal distribution $N(0, 1)$. When both firm i and firm j use frame B, suppose ϵ_i, ϵ_j follow a joint normal distribution with correlation coefficient $\rho \in (-1, 1)$. Then $\epsilon_i - \epsilon_j$ follows a normal distribution with zero mean and variance $2(1 - \rho)$.

When both firms adopt frame A, demand is perfectly elastic at $p_i = p_j$. When only one firm, say, firm i , adopts frame B, its demand function is

$$Q_i = \frac{1}{2} \left(1 - \frac{p_i - p_j}{\alpha_i} \right) \mathbb{1}_{p_i < p_j}$$

However, an interpretation with rational consumers might be inconsistent with the separating equilibrium in Proposition 2 (where the complex frame is always associated with higher prices than the simple one). Rational consumers should be able to infer prices from frames and always choose the simple-frame product.²⁷ Then, the separating equilibrium would not be valid. (This is not an issue in our model with boundedly rational consumers.) Nevertheless, notice that the separating equilibrium could still make sense if there is always a non-trivial mass of naive consumers who do not try to understand market equilibrium.

Carlin (2009) considers a setting related to our case with $\alpha_2 > \alpha_1$. In his model, if a consumer incurs a cost, she can learn all prices in the market, thereby purchasing the cheapest product; otherwise, she remains uninformed and shops randomly. In equilibrium, higher complexity is associated with higher prices. Consumers in Carlin's model cannot infer prices from a firm's price complexity level because they cannot observe individual firms' complexities but only observe the aggregate market complexity.

Dependence between the two sources of confusion. In our oligopoly model in Section 3, we assumed that confusion due to frame differentiation and confusion due to frame complexity are independent and considered up to four types of consumer groups whose sizes are determined by the parameters α_1 and α_2 . However, the two sources of confusion may be dependent. Take, for instance, our numeracy-skill example for $\alpha_1 > \alpha_2$ in subsection 2.1. There, confusion stems from poor numeracy skills and it is mitigated by similarity. So if a consumer is confused by two complex frames, she must also be confused by two different frames.

To allow for dependence between the two sources of confusion, we can regard the four consumers groups as the primitives of the model. A fraction β_{FD} of consumers are confused only by frame differentiation, a fraction β_{FC} of consumers are confused only by frame complexity, a fraction β_B are confused by either source, and the remaining fraction $1 - \beta_{FD} - \beta_{FC} - \beta_B$ of consumers are fully aware. (Note that the two confusion sources are independent if and only if $\beta_{FD} = \alpha_1 - \alpha_2$, $\beta_{FC} = \alpha_2 - \alpha_1$ and $\beta_B = \alpha_1 \alpha_2$.) Then, our analysis carries over with some change of notation.²⁸ In particular, the case with α_1

frames and prices, and make positive profits. An increase in the number of firms reduces firms' ability to frame differentiate and makes them use complex frames more often. As a result, greater competition might increase profits and harm consumers. In our setting, consumer confusion may stem from price format incompatibility or price complexity. The nature of the

A Appendix: Proofs in the Duopoly Case

Proof of Lemma 3: Suppose that F

that should be equal to the candidate equilibrium price. As the supposition $p_1^A < p_1^B$ and Step 1 imply that $p_1^A \in S_B$, the indifference condition requires $A; p_1^A = B; p_1^A$ or

$$- (1 - \alpha) - \alpha = - (1 - \alpha) - \alpha x_B p_1^A :$$

But, if this equation holds, $A; p > B; p$ for $p \in p_1^A$; as $\alpha > 0$ and x_B is strictly decreasing on S_B . A contradiction. Similarly, we can exclude the possibility of $p_1^B < p_1^A$. Hence, it must be that $p_1^A = p_1^B$.

Then, from $A; p_1^A = B; p_1^A$, it follows that

$$1 - \alpha = 1 - \alpha : \tag{16}$$

Now suppose $p_0^A < p_0^B$. Then

$$\begin{aligned} A; p_0^B &= p_0^B x_A p_0^B - (1 - \alpha) = \text{and} \\ B; p_0^B &= p_0^B \{ - (1 - \alpha) \} \text{ given imply that } - p_0^B \end{aligned}$$

Since the supposition $p_0^A < p_0^B$ and Step 1 imply that $p_0^B \in S_A$, we need $A; p_0^B = B; p_0^B$, or

$$x_A p_0^B = \frac{1 - \alpha}{1} :$$

The left-hand side is strictly lower than given th357(nee)-1(d)]TJ/F727d326.216464.85cmq40Td[(2)]-222()-222

Step 1: $S_A \cap S_B = \{p\}$ for some p . Suppose to the contrary that $S_A \cap S_B = [p^0; p^{00}]$ with $p^0 < p^{00}$. Then for any $p \in [p^0; p^{00}]$, it must be that $F_A(p) = F_B(p)$, where the profit functions are given by (2) and (3). This indifference condition requires that

$$\alpha_1 x_A(p) - \beta_1 = \alpha_2 x_B(p) - \beta_2$$

for all $p \in [p^0; p^{00}]$. Since $\alpha_1 < \alpha_2$ and F_Z is strictly increasing on S_Z , the left-hand side is a decreasing function of p ; while the right-hand side is an increasing function of p . So the condition cannot hold for all $p \in [p^0; p^{00}]$. A contradiction.

Step 2: $p_1^B < p_1^A$. Suppose $p_1^B < p_1^A$. Then Step 1 and

Second, each B firm must also earn at least π_{n-1} . Otherwise, any B firm that earns $\pi_B < \pi_{n-1}$ can improve its profit by deviating to frame A and a price $p - \epsilon$ for small ϵ . (The deviator would make a profit at least equal to $p - \epsilon - \pi_{n-2}$ which is greater than π_B for a sufficiently small ϵ given that $\pi_{n-2} \geq \pi_{n-1}$.) Then, if $\pi_{n-1} > \pi_n$, the sum of all firms' profits exceeds one, and we reached a contradiction since industry profit is bounded by one. The only remaining possibility is that $\pi_{n-1} = \pi_n$ and each firm earns exactly π_n . But, then all firms charge the monopoly price p ³² and any B firm has incentives to deviate to a price slightly below one given that $\pi_2 < \pi_n$. A contradiction. ■

Equilibrium condition for π when $\pi_2 < \pi_1$:

Since the price distributions for frames A and B

Proof. At equilibrium, each firm's demand can be decomposed in two parts: the consumers who are insensitive to its price, and the consumers who are price-sensitive. Explicitly, we have

$$A; p = p_0; \quad A; \quad \left\{ \sum_{k=1}^{n-1} x_A p^{n-1-k} + \sum_{k=1}^n P_{n-1}^k x_A p^{n-k} \right\} \text{ and}$$

$$B; p = p_0; \quad B; \quad \left\{ \sum_{k=1}^{n-1} x_B p^{n-1-k} + \sum_{k=1}^n P_{n-1}^k x_B p^{n-k} \right\} :$$

Suppose $x_A(p) = x_B(p) = x(p)$, and the common support is $p_0; \dots$. At equilibrium, $A; p = B; p$ must hold for any $p \in p_0; \dots$.

(i) For $n = 1$, the last term in each demand function disappears. To have $A; p = B; p$ for any $p \in p_0; \dots$, we need $A; \dots = B; \dots$, or equivalently $\frac{1 - \alpha^2}{1 - \alpha} = \frac{1 - \beta^2}{1 - \beta}$; and $\alpha = \beta$, or equivalently $\frac{1 - \alpha^2}{1 - \alpha} = \frac{1 - \beta^2}{1 - \beta}$. It follows that these two conditions hold simultaneously if and only if $\alpha = \beta$.

(ii) With $n \geq 2$, to have $A; p = B; p$ for any $p \in p_0; \dots$, we need $A; \dots = B; \dots$ (see (17)), and

$$\sum_{k=1}^{n-1} x p^{n-1-k} + \sum_{k=1}^n P_{n-1}^k x p^{n-k} = \sum_{k=1}^{n-1} x p^{n-1-k} + \sum_{k=1}^n P_{n-1}^k x p^{n-k} :$$

(To derive the latter, we divided each side by $x p^{n-1}$ and relabelled k in $A; p$ by $n-k-1$.)

Then

$$\sum_{k=1}^n b_k x p^{n-k} = \sum_{k=1}^{n-1} x p^{n-1-k} + \sum_{k=1}^n P_{n-1}^k x p^{n-k} \quad (18)$$

where $b_k \equiv P_{n-1}^{n-k} - P_{n-1}^{n-k-1}$. Since the left-hand side of (18) is a polynomial of $x p$ and $x p$ is a decreasing function, (18) holds for all $p \in p_0; \dots$ only if $b_k = 0$ for $k = 1, \dots, n-1$ and the right-hand side is also zero. That is,

$$\frac{1 - \alpha^{n-1}}{1 - \alpha} = \frac{1 - \beta^{n-1}}{1 - \beta} \text{ and} \quad (19)$$

$$\frac{1 - \alpha^{2k-1}}{1 - \alpha} = \frac{1 - \beta^{2k-1}}{1 - \beta} \text{ for } k = 1, \dots, n-1 : \quad (20)$$

If $\alpha = \beta$, both of them and (17) hold for $\alpha = \beta$ (in which case, $\alpha = \beta$). Beyond this special case, (20) pins down a decreasing sequence $\{\alpha_k\}_{k=1}^{n-1}$ uniquely. Substituting (19) and (20) into (17), we can solve for α_{n-1} . This means that, if $n \geq 2$ and $\alpha_2 > \alpha_1$, price-frame independence can hold only for a particular sequence of α_k .³³ It is easy to verify that $\alpha_k = \alpha$ does not satisfy these conditions. ■

³³Note that, although $\{\alpha_k\}_{k=1}^{n-1}$ solved from (20) is a decreasing sequence, still $\alpha_1 > \alpha_2$, which is solved from (17), may not be lower than α_2 . For example, when $n = 2$, one can check that

$$\alpha_1 = \frac{1 - \alpha^2}{1 - \alpha} < \alpha_2 = \frac{1 - \alpha^2}{\sqrt{1 - \alpha^2}}$$

which violates the requirement that α_k is non-increasing in k .

Proof of Proposition 4: When frame B is also a simple frame (i.e., when $\alpha_2 = 1$), the equilibrium condition (17) for α_1 becomes $\frac{2 - \alpha_1}{n} = \frac{1}{n - \alpha_1}$: It follows that α_1 tends to $\alpha_1^* = 2 - \alpha_2$ as $n \rightarrow \infty$.³⁴ Then industry profit $\pi = \frac{1}{n} \sum_{k=1}^n C_{n-1}^k$ must converge to zero.³⁵

Now consider $\alpha_2 > 1$. Since the left-hand side of (17) is bounded, it must be that $\alpha_1 \leq \alpha_2$ (otherwise the right-hand side would tend to infinity). Since $\{C_{n-1}^k\}_{k=1}^n$ is a non-increasing sequence, the right-hand side of (17) is greater than

$$\frac{2 - \alpha_1}{n} \sum_{k=1}^n C_{n-1}^k \geq \frac{2 - \alpha_1}{n} \frac{1}{n - \alpha_1} = \frac{2 - \alpha_1}{n(n - \alpha_1)} :$$

So it must be that $\alpha_1 \leq \alpha_2 - \frac{1}{n}$, otherwise the right-hand side of (17) tends to infinity (given that $\alpha_1 \leq \alpha_2$ and so $\alpha_1 - \alpha_2 \leq 0$). This result implies that α_1 must converge to zero and industry profit $\pi = \frac{1}{n} \sum_{k=1}^n C_{n-1}^k$

Since the left-hand side of (21) is π , we can solve

$$k_1 = \frac{n}{n-1} k_2 = k_1 - \frac{n^2 - 1}{n-1} k_1^2 :$$

As k_1 decreases with n , π must decrease with n .

As $\pi \approx \frac{1}{n}$ (so that $\pi \approx \frac{1}{n}$), industry profit (for $n \geq 2$)

price). (i) Suppose that, at equilibrium, $p_A > \{ \frac{1}{B} \}$. Then, if the B firm which earns the least deviates to frame A and a price $p_A - \epsilon$, it will replace the original A firm and have a demand at least equal to the original A firm's demand since it now charges a lower price and faces fewer competitors.³⁶ So, this deviation is profitable at least when ϵ is close to zero. A contradiction. (ii) Suppose now that, at equilibrium, $p_A \leq \{ \frac{1}{B} \}$. Notice that $p_A \geq \epsilon$, otherwise the A firm would deviate to frame B and a price p

The equilibrium condition $B; p \in p_0^A; p$ pins down a well-defined α :

$$\frac{\alpha - n}{1} = \sum_{k=1}^{n-1} \frac{\alpha^k C_{n-1}^k}{n-k} \alpha^k \quad (27)$$

The left-hand side of (27) is positive given that $\alpha \geq n$, and the right-hand side is increasing in α from zero to infinity. Hence, for any given $n \geq 2$ and $\alpha \in \mathbb{R}_+$, equation (27) has a unique solution in α .

To complete the proof of Proposition 6, we only need to rule out profitable deviations from the proposed equilibrium. First, consider two possible deviations with frame A: (i) a deviation to $A; p < p_0^A$ is not profitable as the firm does not gain market share, but loses on prices; (ii) a deviation to $A; p > p_0^A$ is not profitable either, since the deviator's profit is $\alpha - n - 1 < 0$.

Let us now consider a deviation to $B; p \in p_0^A$. Deviator's profit is

$$B; p = p_0^A; p = \sum_{k=1}^{n-1} P_{n-1}^k \alpha^k p^k$$

This expression captures the fact that when $n - k$ other firms also use B, or when $k \geq 1$ firms use A and the consumer is confused between A and B, firm i's profit is $\alpha^k p^k$. (Not;(recJ/F72 -333)that, -333reomk52 31

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