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MODELLING VOLATILITY OF CRYPTOCURRENCIES USING MARKOV-SWITCHING GARCH MODELS

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1. Introduction

Modelling volatility is crucial for risk management. Following the global financial crisis of 2008, the Basel III international regulatory framework for banks has imposed more stringent capital requirements, and enhanced risk management systems have been developed. Since then the international financial system has had to face a new challenge, namely the introduction of decentralised cryptocurrencies, the first being Bitcoin, which was created in 2009 (Nakamoto, 2009). Unlike traditional currencies, cryptocurrencies are based on cryptographic proof, which provides many advantages over traditional payment methods (such as credit cards) including high liquidity, lower transaction costs, and anonymity (these features are discussed by Fantazzini et al., 2016).

Interest in Bitcoin and other cryptocurrencies has risen considerably in recent years. Their market capitalisation increased from approximately 18 billion US dollars at the beginning of 2017 to nearly 600 billion at the end of that year[1], and high returns have attracted new investors. In addition, two big exchanges, i.e. the Chicago Mercantile Exchange (CME) and the Chicago Board Options Exchange (CBOE), started to trade futures on Bitcoin.[2] As a result of these developments, central banks have been facing the question of whether or not cryptocurrencies should be regulated, given the numerous technical and legal issues involved.

Further, cryptocurrencies are highly volatile and consequently it is important to estimate appropriate risk metrics, which can be used for calculating capital requirements, margins, hedging and pricing derivatives etc. It is well known that standard GARCH models can produce biased results if the series display structural breaks (Bauwens et al. (2010, 2014)); these are likely to occur in the case of cryptocurrencies. Specifically, Chu et al. (2017) argue that structural changes not accounted for (such as policy changes) might explain why the Integrated GARCH (1, 1) (IGARCH) model is found to be a good fit for numerous cryptocurrencies (Caporal tion costs, and anonymity is then chosen by backtesting VaR and ES as well as using a Model Confidence Set (MCS) procedure for their loss functions.

The paper is organised as follows. Section 2 briefly discusses the relevant literature; Section 3 provides a description of the data; Section 4 outlines the methodology; Section 5 presents the empirical results; finally, Section 6 offers some concluding remarks.

2. Literature Review

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are the most commonly used in the literature for modelling volatility and estimating Value-at-Risk (VaR) and Expected Shortfall

Figure 2 Histograms of log returns

4. Methodology

4.1 GARCH models

Let $y_t \in \mathbb{R}$ be the percentage log-returns of the financial asset (exchange rate) of interest at time *t*. Following Ardia et al. (2018a), we adopt the following general Markov-Switching GARCH specification:

$$y_t \mid (s_t = k, \mathcal{I}_{t-1}) \sim \mathcal{D}(0, h_{k,t}, \xi_k) \qquad (1)$$

where $\mathcal{D}(0, h_{k,t}, \xi_k)$ is a continuous distribution with zero mean, time

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(3)

EGARCH (Nelson (1991))

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4.3 Expected Shortfall Backtesting

VaR is an elicitable

A Wald statistic is then carried out using the parameters ︱ᚶᚿ :

"Y∘ ﯾ ﺓ ︱ᚶᚿ πᚶρ ɫ ﯿ ︱ᚶᚿ πᚶρ ᚶ (31)

where ɫ ﯿ is an estimator for the (asymptotic) covariance matrix of the M-estimator of the parameters ︱ᚶᚿ . Hence, the test statistics follows asymptotically a …⁚ distribution.

The test described above has been named the bivariate ESR test by Bayer and

In order to eliminate inferior elements of the set M^0 the following EPA hypotheses are tested

$$
\begin{aligned}
H_{0,M} &: E(d_{ij,t}) = 0 \quad \text{for all } i,j \in M \\
H_{A,M} &: E(d_{ij,t}) \neq 0 \quad \text{for some } i,j \in M
\end{aligned}
\tag{38}
$$

or

$$
\begin{aligned}
H_{0,M} &: E(d_{i\cdot,t}) = 0 \quad \text{for all } i \in M \\
H_{A,M} &: E(d_{i\cdot,t}) \neq 0 \quad \text{for some } i \in M
\end{aligned}
\tag{39}
$$

where $M \subset M^0$.

From these statistics the following *t*-statistics can be constructed

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{\widehat{\operatorname{var}}(\bar{d}_{ij})}} \quad \text{and} \quad t_{i\cdot} = \frac{\bar{d}_{i\cdot}}{\sqrt{\widehat{\operatorname{var}}(\bar{d}_{i\cdot})}}$$

Standard GARCH models prevail in the first and second regimes. Interestingly, the normal distribution prevails in the first regime, but whilst the Student's t distribution is appropriate for 70% of the models in the second regim

second standard GARCH and GJRGRACH represent 90% of the chosen models. Similarly, the Student's t and normal distribution prevail in the second regime being selected in 88% of the cases. No models with a skewed GED distribution are included in the SSM, and none with a skewed distribution skewed Student's t distributions are chosen in all cases in the second. Markov-switching and mixture models represent respectively 60% and 40% of those included in the SSM. Specifications with leverage effects at least in one of the regimes represent 80% of the total in the SSM.

Table 8

namely Bitcoin, Ethereum, Ripple and Litecoin. Two-regime GARCH models are found to produce better VaR and ES predictions than single-regime models.

Surprisingly, both the QL and joint loss function results suggest a standard GARCH model for Bitcoin. As for Ethereum, the MCS procedure selects a regime mixture model with a GJR GARCH specification and Student's t

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