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Antonios Antypas, Guglielmo Maria Caporale, Nikolaos Kourogenis and Nikitas Pittis

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Estimation of conditional asset pricing models with integrated variables in the beta speci cation

Antonios Antypas[1,a], Guglielmo Maria Caporale[b,c,d], Nikolaos Kourogenis[a,], Nikitas Pittis[a]

[a]Department of Banking and Financial Management, University of Piraeus, Greece
[b]Department of Economics and Finance, Brunel University London, UK
[c]CESifo, Munich, Germany
[d]DIW Berlin, Germany

Abstract

We introduce a methodology which deals with possibly integrated variables in the speci cation of the betas of conditional asset pricing models. In such a case, any model which is directly derived by a polynomial approximation of the functional form of the conditional beta will inherit a nonstationary right hand side. Our approach uses the cointegrating relationships between the integrated variables in order to maintain the stationarity of the right hand side of the estimated model, thus, avoiding the issues that arise in the case of an unbalanced regression. We present an example where our methodology is applied to the returns of funds{of{funds which are based on the Morningstar mutual fund ranking system. The results provide evidence

Corresponding author:
Nikolaos Kourogenis, Department of Banking and Financial Managenent, University of Piraeus. 80 Karaoli & Dimitriou str., 18534 Piraeus, Greece.
Email addresses: ant_antypas@yahoo.com (Antonios Antypas), Guglielmo-Maria.Caporale@brunel.ac.uk (Guglielmo Maria Caporale), nkourogenis@yahoo.com and nkourog@unipi.gr (Nikolaos Kourogenis), npittis@unipi.gr (Nikitas Pittis)

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that the residuals of possible cointegrating relationships between integrated variables in the specification of the conditional betas may reveal significant information concerning the dynamics of the betas.
Keywords: Conditional CAPM, Time-varying beta, Cointegration, Morningstar star-rating system
JEL: G1, G23, C1

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1. Introduction

The Capital Asset Pricing Model (CAPM), proposed by Treynor (1962), Sharpe (1964) and Lintner (1965), has been a cornerstone of the modern asset pricing theory. This model postulates that the expected excess return, $E(R_j - R_f)$, on asset j, (that is, the expected return R_j minus the known risk-free rate R_f) is linearly related to the 'beta', β_j, of asset j, $\beta_j \equiv Cov(R_j, R_m)/Var(R_m)$ where R_m denotes the returns on the market portfolio and $E(R_j - R_f) = \beta_j E(R_m - R_f)$. Since the inception of CAPM, numerous asset pricing models have been developed, such as the Single Factor Model and its multivariate generalization, the Multiple Factor Model, the Arbitrage Pricing Model (Ross (1976)) and the Intertemporal Capital Asset Pricing Model (Merton (1973)). These models have inspired the development of a large number of variations or extensions.

One common assumption used in the aforementioned models, is that of linear relationships between the (excess) return of asset j and the corresponding risk factors. For the estimation of these models, it was initially assumed that the slope coefficients (the betas) remain constant over time, or over the estimation window. There is, however, overwhelming evidence suggesting

assumption imposes restrictions on the selection of the variables included in Z_t so that the right hand side of the approximate model remains stationary. For example, if $n = 1$ and Z_t is I(1), then we will face the problem of an unbalanced regression because both conditional and unconditional variances of the right hand side of the model will be explosive. In general, this may be the case if some (or all) of the variables in Z_t are integrated.

In the next section we propose a methodology that allows us to exploit possible relationships between integrated variables, so that their inclusion in Z_t does not violate the stationarity requirement. In the third section we apply our methodology to the returns of funds-of-funds which are created with respect to the star-rating system of Morningstar. The fourth section concludes the paper.

2. Integrated variables in the beta speci cation

As pointed out in the previous section, the inclusion of integrated variables in the speci cation of the conditional betas can lead us to spurious conclusions. In this section we present a methodology for the appropriate treatment of these variables. For expository simplicity we use a simple one factor model. Our methodology, however, can be directly extended to multiple factor models because the approximations of the functional forms of each beta are treated separately.

2.1. Formalization and treatment of the problem

Let us begin by using a rst order approximation of b_j in equation (1). We obtain

$$r_{j;t+1} = {}_{j;0} + {}^{c}_{j} r_{m;t+1} + r_{m;t+1} \sum_{i=1}^{n} {}_{j;i} Z_{i;t} + u_{j;t+1} : \qquad (4)$$

It is quite natural to assume that both $r_{j;t+1}$ and $r_{m;t+1}$ in (4) are stationary. On the other hand, this cannot be a priori assumed for $Z_{1;t}; Z_{2;t}; :::; Z_{n;t}$. Therefore, we must rst analyze in detail the alternative models (all based on (4)) that arise depending on the statistical properties of the variables $Z_{1;t}; Z_{2;t}; :::; Z_{n;t}$: Speci cally, we distinguish three cases:

(i) The vector stochastic process $[r_{m;t+1}; Z_t^0]^0$ is stationary. This implies ompympto]TJ/F9(Som5-1uo)-275.2 that the new variables $Z_{i;t} r_{m;t+1}$ will also be I(0) (given that the market returns variable $r_{m;t+1}$ is quite naturally I(0)), and equation (4) is legitimate since $r_{j;t+1}$ is also quite naturally I(0)).

(ii) Some (or all) of the variables $Z_{1;t}; Z_{2;t}; :::; Z_{n_0;t}; n_0$ n are I(1) and not cointegrated. In this case, the product variables $Z_{i;t} r_{m;t+1}; i = 1; 2; :::; n^0$ stocha68stocha5+1 on the right-hand side of (4). The following example highlights differences between case (ii) and case (iii):

Let

$$r_{j,t+1} = \beta_{j,0} + \beta_j^c r_{m,t+1} + (\beta_{j,1} Z_{1,t} + \beta_{j,2} Z_{2,t})\, r_{m,t+1} + u_{j,t+1} \qquad (5)$$

If $Z_{1,t}$ and $Z_{2,t}$ are not cointegrated and $\beta_{j,1}\beta_{j,2} \neq 0$, the unconditional variance of the right-hand side will grow to infinity as $t \to \infty$, violating our initial assumption that the unconditional variance of $r_{j,t+1}$ is bounded. Therefore the estimated values of $\beta_{j,1}$ and $\beta_{j,2}$ will be very close to 0 when the sample is large. If, on the other hand, $Z_{1,t}$ and $Z_{2,t}$ are cointegrated and satisfy

$$Z_{1,t} = a_0 + a_1 Z_{2,t} + w_t$$

where $\{w_t\}_{t \geq 1}$ is $I(0)$, the only way for the unconditional variance of $\beta_{j,1} Z_{1,t} + \beta_{j,2} Z_{2,t}$ to remain asymptotically bounded, with nonzero $\beta_{j,1}$ and $\beta_{j,2}$, is the case where

$$\beta_{j,1} Z_{1,t} + \beta_{j,2} Z_{2,t} = \gamma(Z_{1,t} - a_1 Z_{2,t})\, r_{m,t+1} = \gamma(a_0 + w_t)\, r^2$$

order to obtain in (4) a well-balanced regression. Specifically, the following steps must be taken:

First, we identify all the state variables (elements of Z_t) that are $I(1)$. Assume that the number of such $I(1)$ variables is n_0. If $n_0 > 0$, without loss of generality, reordering the variables if necessary, we can make sure that, for $i \le n_0$, $\{Z_{i,t}\}_{t \ge 1}$ are $I(1)$ and, for $n_0 < i \le n$, $\{Z_{i,t}\}_{t \ge 1}$ are $I(0)$. Second, we identify any cointegrating relationships between the processes $\{Z_{i,t}\}_{t \ge 1}$, $1 \le i \le n_0$. Let $k < n_0$ be the rank of the cointegrating system. This means that we can find a $(k \times n_0)$ matrix A of order k, such that

$$A\,[Z_{1,t}, Z_{2,t}, \cdots, Z_{n_0,t}]' = U_t\,, \tag{7}$$

where $\{U_t\}_{t \ge 0}$ is $I(0)$ with nontrivial coordinates $U_{i,t}$, $1 \le i \le k$. Again, without any loss of generality, we can reorder the variables $Z_{i,t}$, $1 \le i \le n_0$ in (4), so that the first k columns of A are linearly independent. Therefore, we can write $A = [A_1, A_2]$, where the $k \times k$ matrix A_1 is invertible. Then, left multiplication of (7) by A_1^{-1} yields

I_k

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$Z_{i;t}$, $1 \le i \le n_0$. Third, having defined the appropriate set, W_t; of I(0) state variables,

$$W_t = (W_{i;t})_{1 \le i \le n} = [U_{1;t}; U_{2;t}; \ldots; U_{k;t}; \Delta Z_{k+1;t}; \ldots; \Delta Z_{n_0;t}; Z_{n_0+1;t}; \ldots; Z_{n;t}]^0$$

we can run the following regression

$$r_{j;t+1} = \beta_{j;0} +$$

The last equation involves $n + \binom{n}{2} = \frac{n(n+1)}{2}$, d explanatory variables of the form $W_{i;t} r_{m;t+1}$ or $W_{i;t} W_{k;t} r_{m;t+1}$, $1 \quad i \quad k \quad n$, which can be denoted as $X_{l;t+1}$, $1 \quad l \quad d$.

We can rewrite (14) as:

$$r_{j;t} = b_{j;0} + {}^{c}_{j} r_{m;t+1} + \sum_{i=1}^{d} b_{j;i} X_{i;t} + {}_{j;t} \qquad (15)$$

where $b_{j;0} = {}_{j;0}$, $b_{j;i} = {}_{j;i}$ and $X_{i;t} = W_{i;t\ 1} r_{m;t}$ for $1 \quad i \quad n$, $b_{j;i} = {}_{j;g;h}$ and $X_{i;t}$ is of the form $W_{g;t\ 1} W_{h;t\ 1} r_{m;t}$

Table 1: Explosive behavior of a returns process when the conditional beta is not stationary.

Sample size	Average s.d.(y_1)	Average s.d.(y_2)
50	1.11	2.48
100	1.11	3.36
500	1.12	7.03
1000	1.12	9.86
2000	1.12	13.93
5000	1.12	21.81
10000	1.12	30.94

using portfolios of funds (funds-of-funds) which are based on the Morningstar star-rating system.

3. An application to funds{of{funds based on the Morningstar star-rating system

This section aims to demonstrate how the methodology presented in the previous section can be implemented. To this end, we use Morningstar fund data, aiming to identify possible factors that a fund manager may consider when she/he adjusts the exposure of the corresponding portfolio to systematic risk (the portfolio's beta). The data used in our study were taken from Morningstar Direct, which provides historical monthly returns of mutual funds along with their star-based rankings calculated by Morningstar. The Morningstar 'star-rating' is a risk-adjusted performance measure, which ranges from one-star to five-stars (higher rating implying better risk-adjusted performance), and usually varies slowly over time.

In 2002, Morningstar modified the star rating system by introducing new peer groups and a new measure of risk-adjusted return (see Fuss et al. (2010) for an informative brief description of the procedure used by Morningstar). This fact implied that incorporating data prior to July 2002 would add a significant heterogeneity factor in our sample. In order to avoid possible implications of this structural change, we cover the subsequent period (July 2002 - September 2018). We focus on the subset represented by US domiciled equity mutual funds that invest at least 90% of their Non-cash Adjusted Total Assets in equity securities around the world. To avoid dealing with currency risk exposure we only consider funds quoted in US Dollars.

Given the availability of the star-ratings, we proceed further in our inaccount for the expense ratio, which includes management, administrative, 12b-1 fees, and other costs that are taken out of assets." Thus, starting from the Total Returns and reversing the effect of the expense ratio, Morningstar calculates the Gross Returns. This returns series provides us a clearer performance from the fund manager perspective, because it is directly related to the performance of the constituents of the fund, and their corresponding weights in the allocation scheme that the fund manager has chosen.

In order to create the portfolios and the corresponding returns we proceed as follows: The first portfolio, named STAR1, consists of all the funds that in each time period, t, are rated one-star by Morningstar. Specifically, in period $t = 1$ (the first period in our sample) we invest an amount A$ in a portfolio consisting of all the funds (equally weighted) that have been given one-star by Morningstar in period $t = 1$. In period $t = 2$, the amount $(1 + R_1^1)$ A (R_1^1 being the return of the portfolio between periods 1 and 2) is invested again in a portfolio consisting solely of funds that in period 2 were rated one-star by Morningstar. We continue this process until we reach period $t = T$, i.e. the last period of our sample. In this way, we obtain a series of returns $R_1^1; R_2^1; \ldots; R_T^1$ generated by investing exclusively in one-star funds. These are interpreted as being a random vector from the process $\{R_t^1\}$ generating one-star portfolio returns. We repeat the same procedure for two-, three-, four- and five-star funds, thus obtaining samples from the returns processes $\{R_t^2\}$, $\{R_t^3\}$, $\{R_t^4\}$, $\{R_t^5\}$, which are supposed to generate returns for the two-, three-, four- and five-star funds respectively. Finally, following how an investor acts when a fund stops performing, we exclude non-surviving funds from the allocation procedure after their exit of the Morningstar rating system.

3.1. Identification of significant factors in the dynamics of conditional betas

We consider an asset pricing model that describes the relationship between the expected return and risk of the various portfolios under consideration. Specifically, we adopt the conditional CAPM model of Ferson and Schadt (1996) and Shanken (1990) in which the level of the systematic portfolio risk is a function of the observed variables (see also, Lettau and Ludvigson (2001)). This in turn implies that the relationship between the excess returns of the portfolio j and the excess returns of the market factor is given by the relationships (1)-(3), where, now, $r_{j;t} = R_t^j - R_{f;t}$; $i = 1; 2; ::: ; 5$, $R_{f;t}$ is the return of a one-month Treasury bill, $Z_t = [Z_{1;t}; Z_{2;t}; : : : ; Z_{n;t}]$ is an n-vector of state variables observable by the managers at time t, and $r_{m;t} = R_t^m - R_{f;t}$ where R_t^m stands for the returns of the market factor (returns on the S&P 500). This specification implies that the systematic risk of the portfolio j, as measured by $b_j(Z_t)$, changes with time.

The time-varying nature of beta is due to the fact that the portfolio manager receives at time t the extent to which he/she can translate the information content of Z_t into predictions on the future behavior of $r_{m;t+1}$: This does not necessarily mean that \everybody" in the market can \read" the information contained in Z_t. In other words, although the variables Z_t are indeed publicly available, the information content of Z_t might be available only to a 'skillful' fund manager.

As far as the selection of the variables in Z_t is concerned, we follow Ferson and Schadt (1996), by including the 1-month Treasury bill yield, z_{1t}, the term spread, z_{2t}, defined as the difference between the constant-maturity 10-year Treasury bond yield and the 3-month Treasury bill, the quality spread in the corporate bond market, z_{6t}, defined as the Moody's BAA-rated corporate bond yield minus the AAA-rated corporate bond yield, the S&P 500 dividend yield, z_{8t}, and a dummy variable, z_{9t}, for the January effect. In addition, we include variables that are usually considered important indicators by the financial community such as the price of oil, z_{3t}, the weighted average of the foreign exchange value of the US dollar against a subset of the broad index currencies, z_{4t}, the Consumer Sentiment Index of the University of Michigan, z_{5t}, and the Chicago Board Options Exchange volatility index (VIX), z_{7t}.[2]

Finally, note that the above model can be augmented by the market timing term, $_j r^2_{m;t+1}$; proposed by Treynor and Mazui (1966). A positive (negative) timing coefficient $_j$ is interpreted as evidence suggesting superior (inferior) market timing abilities of the corresponding fund manager.

[2]The source of z_{1t}, z_{2t}, z_{5t} and z_{6t} is Bloomberg, of z_{3t}, z_{4t} and z_{7t} is FRED St. Louis, and of z_{7t} is Standard's & Poors.

3.1.1. Time series properties of the state variables

As explained in the previous section, the choice of the appropriate model for conditional portfolio evaluation depends on the statistical properties of the state variables $z_{1t}; z_{2t}; ...; z_{8t}$, considering that z_{9t} is a dummy variable. The results from ADF and Phillips-Peron unit root tests, reported in Table 2A, unambiguously indicate that the first five series are I(1), while $z_{6t}; z_{7t}; z_{8t}$ are I(0). In order to test the existence of cointegration relationships between z_{1t}, z_{2t}, z_{3t}, z_{4t} and z_{5t} we set the lag-length, l, of the Vector Autoregressive model, VAR(l) equal to 4. The results reported in Table 2B show that the the trace (TR) statistic of Johansen (1991) identifies at most one cointegration relationship between the I(1) state variables. On the other hand, under the hypothesis of no cointegration, the maximum eigenvalue (λmax) statistic is slightly below the 5% critical value. As a result, we run two alternative conditional regressions assuming k = 0 and k = 1 (assuming GARCH(1,1) errors and including the market timing term $\gamma_j r^2_{m;t+1}$).

3.1.2. Estimation results

Concerning the identification of the cointegrating relationship when we assumed that k = 1, we proceeded as follows: First, we searched between all combinations of the five I(1) variables by four for the one whose trace statistic has the lowest p-value. We found that the most probable cointegrating relationship involves $z_{1t}; z_{3t}; z_{4t}$ and z_{5t} with a corresponding p-value equal to 0.0511. Since this p-value exceeds 5%, one could claim that all five time series are included in the cointegrating relationship. However, because the p-value is only slightly over 5%, it seems natural to check whether there is

Table 2: Statistical properties of the state variables.

A. Unit root tests

Variable	ADF	P-P	Constant	Trend
z_{1t}	-0.92	-0.93	N	N
z_{2t}	-1.48	-1.48	N	N
z_{3t}	-2.49	-2.21	Y	N
z_{4t}	-2.44	-2.28	Y	N
z_{5t}	-2.46	-2.48	Y	Y
z_{6t}	-3.79***	-3.79***	Y	N
z_{7t}	-3.41**	-2.91**	Y	N
z_{8t}	-3.46**	-2.90**	Y	N

***: p-val< 1% **: 1%, p-val< 5%, *: 5% p-val< 10%

B. Testing for cointegration amzog

evidence of cointegration in a subset of three variables among $z_{1t}; z_{3t}; z_{4t}$ and z_{5t}. We proceed to the next step, which is to identify any possible cointegrating relationships that involve at most three of $z_{1t}; z_{3t}; z_{4t}$ and z_{5t}. Using the same p-value approach, we identify a relationship between $z_{3t}; z_{4t}$ and z_{5t}, with a corresponding p-value equal to 0:0372. Since this p-value is lower than 0:05 we proceeded by searching for a cointegrating relationship between all possible pairs from $z_{3t}; z_{4t}$ and z_{5t}. Then we identify a cointegrating relationship between z_{3t} and z_{4t} with a p-value of 0:0417. We have used this relationship in our estimation, as described in the previous section.

The results presented in Table 3 correspond to the application of a general-to-specific approach, where we start from equation (11) and in each step all terms involving the factors z_i, $1 \le i \le 9$, and having coefficients with a p-value greater or equal than 0.05 are omitted. Then the equation is re-estimated.

Tables 3A and 3B report the results that correspond to the cases of zero

Table 3: Estimation of conditional models for the star-rated funds-of-funds. (GARCH(1,1) error speci cation).

A: Cointegration rank among z_{1t}, z_{2t}, z_{3t}, z_{4t} and z_{5t} equals zero. z_{1t}, z_{2t}, z_{3t}, z_{4t}, z_{5t}, z_{6t}, z_{7t}, z_{8t} and z_{9t} are employed as state variables.

Fund-Of-Funds	a	t(a)		t ( )	Signi cant State Variables (p-value< 0.05)	AIC	SIC
STAR1	0.002	1.47	-0.79	-2.64	$(z_{2t\ 1})$, $z_{8t\ 1}$	-5.79	-5.65
STAR2	0.002	2.50	-0.50	-1.92	$(z_{2t\ 1})$, $z_{8t\ 1}$	-6.04	-5.90
STAR3	0.002	2.86	-0.35	-1.91	-	-6.25	-6.15
STAR4	0.003	3.60	-0.57	-2.16	$(z_{5t\ 1})$	-6.26	-6.14
STAR5	0.003	2.72	-0.51	-1.42	-	-6.00	-5.90

B: Cointegration rank among z_{1t}, z_{2t}, z_{3t}, z_{4t} and z_{5t} equals one. u_{1t}, z_{1t}, z_{2t}, z_{5t}, z_{6t}, z_{7t}, z_{8t} and z_{9t} are employed as state variables, where u_{1t} corresponds to the cointegrating relationship between z_{4t} and z_{5t}.

Fund-Of-Funds	a	t(a)		t ( )	Signi cant State Variables (p-value< 0.05)	AIC	SIC
STAR1	0.002	1.47	-0.787	-2.64	$(z_{2t\ 1})$, $z_{8t\ 1}$	-5.79	-5.65
STAR2	0.003	2.82	-0.726	-2.54	$u_{1t\ 1}$, $(z_{2t\ 1})$	-6.07	-5.93
STAR3	0.003	3.78	-0.803	-2.98	$u_{1t\ 1}$, $(z_{2t\ 1})$, $(z_{5t\ 1})$	-6.29	-6.14
STAR4	0.003	4.01	-0.685	-2.46	$u_{1t\ 1}$, $(z_{5t\ 1})$	-6.28	-6.14
STAR5	0.003	2.72	-0.513	-1.42	-	-6.00	-5.90

C: Regressions based on Ferson and Schadt (1996). z_{1t}, z_{2t}, z_{6t}, z_{8t} and z_{9t} are employed as state variables.

Fund-Of-Funds	a	t(a)		t ( )	Signi cant State Variables (p-value< 0.05)	AIC	SIC
STAR1	0.001	1.14	-0.495	-1.46	$z_{8t\ 1}$	-5.77	-5.66
STAR2	0.002	2.19	-0.237	-0.98	$z_{8t\ 1}$	-6.03	-5.91
STAR3	0.002	2.86	-0.354	-1.91		-6.25	-6.15
STAR4	0.003	3.14	-0.444	-1.93		-6.25	-6.15
STAR5	0.003	2.72	-0.513	-1.42		-6.00	-5.90

transform these variables in order to maintain stationarity of the right hand side of the assumed asset pricing model. The straightforward approach is to replace these variables with their first differences.

Our methodology provides an alternative treatment of the nonstationarity problem of the right hand side. Specifically, our approach makes use of possible cointegrating relationships between the variables in the functional forms of the conditional loadings. We show that by replacing the cointegrated variables with the corresponding residuals of the cointegrating relationship, we maintain the stationarity of the right hand side of the asset pricing model.

Next, we provided an example by applying our methodology to funds–of–funds which are based on the Morningstar mutual fund ranking system (see, e.g., Blake and Morey (2000)). Specifically, we considered a conditional CAPM (see Ferson and Schadt (1996) and Shanken (1990)) in which portfolio risk is a function of observed variables.

We showed that when the approach in Ferson and Schadt (1996) is used, only the dividend yield of the S&P 500 appears to be a statistically significant factor in the specification of the conditional betas. By employing a broader set of candidate state variables for the specification of the betas, we proceeded by considering two cases. The first corresponded to no cointegration between the I(1) variables. Interestingly, however, we identified a possible cointegrating relationship between two of the I(1) variables, namely, the price of oil and the US dollar exchange rate. We showed that the residuals of this relationship appear to be statistically significant factors when they are used in the functional form of the betas. On the other hand, the first differences of both the price of oil and the US dollar exchange rate are not statistically signi cant when no cointegration is assumed.

The methodology presented in Section 2, along the results of the subsequent empirical study support the view that the residuals of cointegrating relationships between integrated variables in the speci cation of the conditional betas may reveal signi cant information about the dynamics of the betas.

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