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Dynamic Factor Models and Fractional Integration ï With an Application to US Real Economic Activity

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1. Introduction

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2. A Review of the Existing Models

2.1 Dynamic Factor Models

The original Stock-Watsonôs (1988) dynamic factor model decomposes the dynamics of a set of time series into a common factor and an idiosyncratic part. With series in first differences and modelled as second-order autoregressive Gaussian processes (2) (Kim & Halbert, 2000) one obtains the following specification:

$$= \quad + \qquad (1)$$

$$= \quad _1 \quad , -1 +$$

- Covariance stationarity if $0 < \quad <$

3 The Proposed Framework

3.1 Model Specification

Our proposed framework introduces fractional integration

4 Data and Empirical Results

4.1 Data Description and Sources

We select the series for the empirical application following the paper by Stock and that are highly negative and p-values that are significantly below 0.05. This suggests that all of them are stationary in first differences. Additionally, the series have ERS test statistics that are below the critical value of 1.99. This indicates that the null hypothesis of a unit root (non-stationarity) can be rejected for all the differenced series.

Table 1. ADF test for the Economic Activity Series

R VR	1 3 a aVaVP	1 3 c R
RR		
R		
Q a V Q PaV		
S Pa V		
R P R		

Computed using the tseries package on R from Trapletti & Hornik (2020)

Table 2. ERS test for the Economic Activity Series

R VR	a aVaVP	VaVP C R
RR		

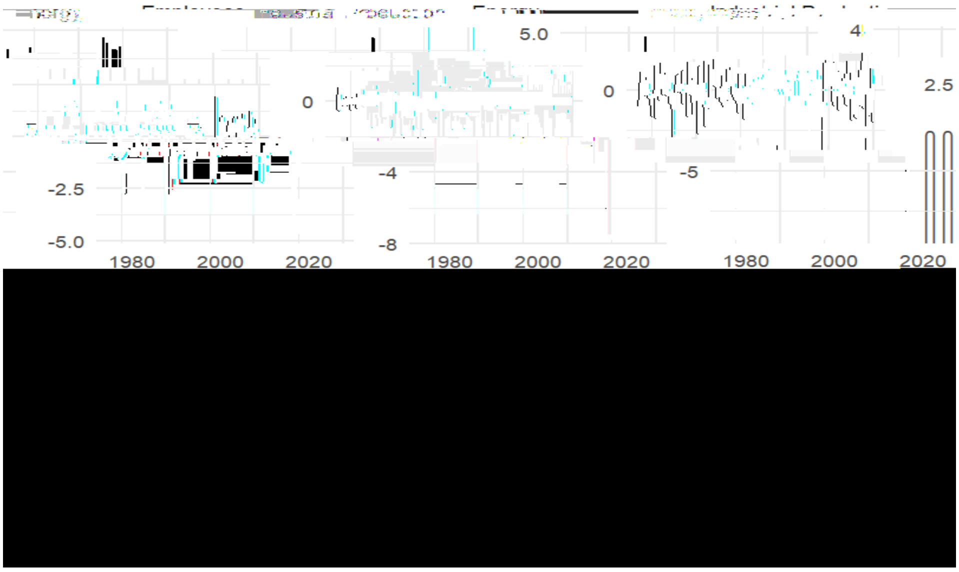
Computed using the urca package on R from Pfaff (2008)

The Shapiro-Wilk (1965) test was then employed to assess the normality of the various economic indicators. The results are displayed in Table 3. The p-values for all economic indicators are 0, which is significantly below the 0.05 threshold. This implies that the null hypothesis of normality is rejected for all series.

Table 3. Saphiro-Wilks test for the Economic Activity Series

Computed using Royston's (1982) algorithm.

Figure 1. Real Activity Variables



Source: FRED (2024). The series depicted in the graph are seasonally adjusted, first differenced, centred around the mean and scaled by the standard deviation.

Figure 1 shows the time series for all five economic indicators, retrieved from FRED (2024) over the period from 1967 to 2019. This period excludes the Covid-19 pandemic and its resulting structural changes, comprising a total of 635 observations.

Table 4. Statistical Summary of the Real Activity Series

Table 5. Statistical summary of the parameter's distributions.

The variables are in the same order as described in the data. The parameters are the autoregressive coefficients of the factor, the ones are the loadings and the ones are the variances of the idiosyncratic disturbance terms. Q stands for quantile and SD for standard deviation.

Figure 2. The Monthly Index of Economic Activity

The monthly index of economic activity shows the median together with the first and third quartile of the factor distribution (dashed). This figure follows Figure 1 in Stock and Watson (1988) and is based on FRED (2024) data.

For the computation of we first allow for a linear time as is common in the unit roots literature (Bhargava, 1986, Schmidt and Phillips, 1992), such that the model becomes a combination of (7) and (8), i.e.,

$$(\ ) = \quad + \quad + (\ ), \qquad (1 - \quad) \quad (\ ) = \quad (\ ), \qquad (6)$$

where α and β are jointly estimated with d, and $u(t)$ follows a white noise process with zero mean and constant variance.

The Lagrange multiplier test for the differencing parameter of the hidden factor are carried out using three different model specifications and under the assumption of white noise residuals; the results can be summarised as follows:

(i) In the first case, we include a constant and a linear trend, and thus and are estimated together with . The test provides the following value and confidence interval for the differencing parameter: $= 2.09\ (2.01, 2.18)$. However, is non-significant, therefore we remove the linear trend.

(ii) In the second case, we allow for a constant but not for a linear trend, namely $= 0$. We then obtain the result $= 2.10\ (2.02, 2.19)$ with $= 0.930$ statistically significant with a t-value of 4.25.

(iii) In the third case, neither a constant nor a trend are included, i.e. $= \quad = 0$ a priori. We obtain the same result as in case (ii), namely $= 2.10\ (2.02, 2.19)$.

Next, we allow for autocorrelation in ( ) and estimate the model using the non-parametric approximation of Bloomfield (1973) for AR structures. The results are now the following:

(i) With a constant and a linear time trend, $= 1.93\ (1.72, 2.16)$. However, the linear trend is statistically insignificant.

5. Conclusions

This paper makes a twofold contribution, First, it develops the dynamic factor model of Barigozzi et al. (2016) by allowing for fractional integration instead of imposing the classical dichotomy between I(0) stationary and I(1) non-stationary series. This more general setup is applicable in a variety of contexts and enables one to consider a much wider range of stochastic processes and to obtain valuable information about the dynamics of the series, such as their degree of persistence and mean reversion. Second, the proposed framework is used to analyse the behaviour of five annual US Real Economic Activity series (Employees, Energy, Industrial Production, Manufacturing, Personal Income) over the period from 1967 to 2019 in order to shed light on their persistence and cyclical behaviour. The results indicate that economic activity in the US is highly persistent and is also characterised by cycles with a periodicity of 6 years and 8 months.

Our findings have important policy implications. Specifically, the evidence that shocks have long-lived effects suggests that they originate from the supply side. It is well known that traditional stabilisation policies have an important role to play in smoothing the amplitude of fluctuations associated with the cyclical behaviour of economic activity and generated by demand shocks (Clarida et al., 1999; Woodford, 2003; Blanchard and Riggi, 2013). By contrast, effective policy responses to supply shocks require structural reforms and investment in productivity-enhancing technologies to achieve sustained growth (Kydland and Prescott, 1982). Given the evidence presented above it appears that it is the latter set of policies that are most appropriate in the case of the US.

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