Trapped fermions and RMT models

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Non-interacting fermions in $d = 1$ at $T = 0$

N non-interacting fermions: $\hat{H}_N = \sum_{n=1}^{N} \hat{H}_n$

Single-particle wave functions

$$\hat{H}\ _k(x) = \frac{1}{2}@_x^2\ _k(x) + V(x)\ _k(x) = \ _k\ _k(x)\ ; \quad \int dx\ _k(x)\ _l(x) = \ _{k;l}$$

At $T = 0$, only occupied levels $_1$ $_2$ $_N$ =

$$j\ _0(x_1; \quad ; x_N)j^2 = \frac{1}{N!}\ _1 \det_{i;j\ N}\ _j(x_i)^2 = \frac{1}{N!}\ _1 \det_{i;j\ N} K\ (x_i; x_j)$$

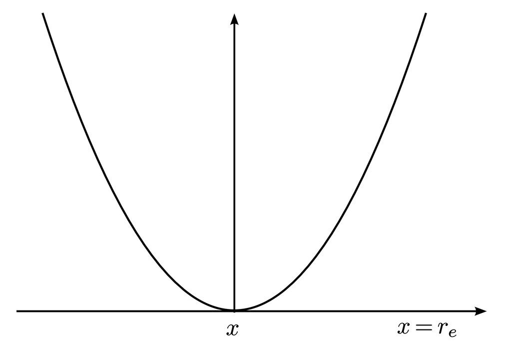
) Determinantal Point Process (valid for any d 1 at $T = 0$) [1]

$$K\ (x; y) = \sum_{k=1}^{N}\ _k(x)\ _k(y) = \sum_k\ _k(x)\ _k(y)\ (\ \ \ _k)$$

Mean density of fermions [1]

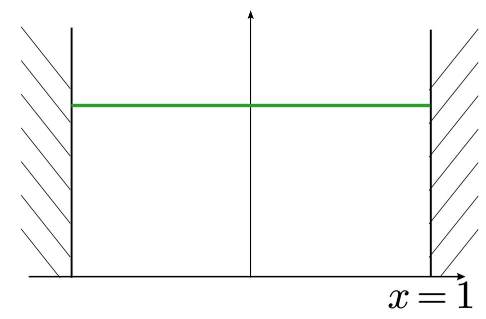
$$\tilde{\ }(x) = \frac{1}{N}h\sum_k (x\ \ x_k)i = \frac{1}{N}K\ (x; x)\ !\ \tilde{\ }^b_1(x) = \frac{1}{\ }\sqrt{2(\ \ V(x))}$$

Soft edges
$V(r_e) =$ for $r_e < 1$
) Continuous $\tilde{\ }^b_1(x)$



VS

Hard edges
$V(R)\ !\ 1$ for $R < 1$
) Disontinuous $\tilde{\ }^b_1(x)$



For specific potentials $V(x)$, mapping with RMT (= 2)

$$j\ _0(x_1; \quad ; x_N)j^2 \quad \underbrace{|}_{\text{Exact mapping}} \quad P_{\text{joint}}(\ _1; \quad ; \ _N) = \frac{1}{Z_N}\prod_{i<j} j\ _i \quad \ _jj^2 \prod_k w(\ _k)$$

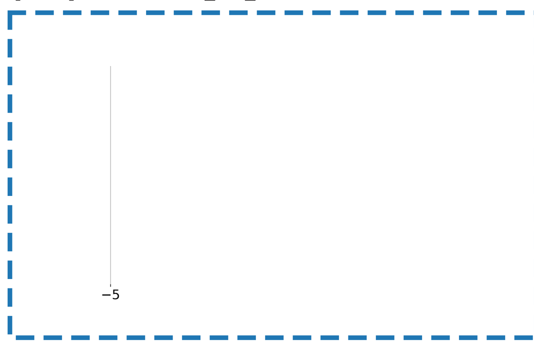
GUE and soft edge

$V(x) = \frac{x^2}{2}$, $w(\ ) = e^{\ \ ^2}$
x_i , $_i$

At the soft edge, typical scale w_N $N^{1=6}$

$$K_1^{\text{soft}}(u; v) = \frac{\text{Ai}(u)\,\text{Ai}^0(v) \quad \text{Ai}^0(u)\,\text{Ai}(v)}{u \quad v}$$

The behaviour is Universal for smooth potentials $V(x)$ x^p with a scale w_N / $V(r_e)$ $^{1=3}$ [1]



LUE and hard edge

$V(x) = \frac{x^2}{2} + \frac{(\ +1)}{2x^2}$; $x > 0$, $w(\ ) =$ e
x_i^2 , $_i$

At the hard edge, = + 1=2

$$K_J(u; v) = \frac{p\ \overline{uv}\ \ uJ_{\ +1}(u)J\ (v) \quad vJ_{\ +1}(v)J\ (u)}{u^2 \quad v^2}$$

(! 1 ! 0)

JUE and hard edge

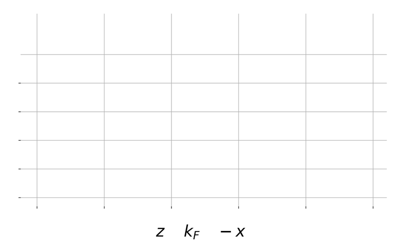
$V(x) = \begin{cases} 0\ ;\ jxj & 1 \\ +1 & \text{otherwise} \end{cases}$, $w(\ ) = \sqrt{(1\ \ \ )}$

$\frac{1}{2} + \frac{1}{2}\sin\ \frac{x_i}{2}$, $_i$

At the hard edge: [2]

$$K_1^{e}(u; v) = \frac{\sin(u \quad v)}{(u \quad v)} \quad \frac{\sin(u+v)}{(u+v)}$$

(see also [3] for other boundary conditions)



Inverse power law potentials

We consider the inverse power law potential $V(x) = \frac{(\ \ +1)}{2x}$ with > 0. At

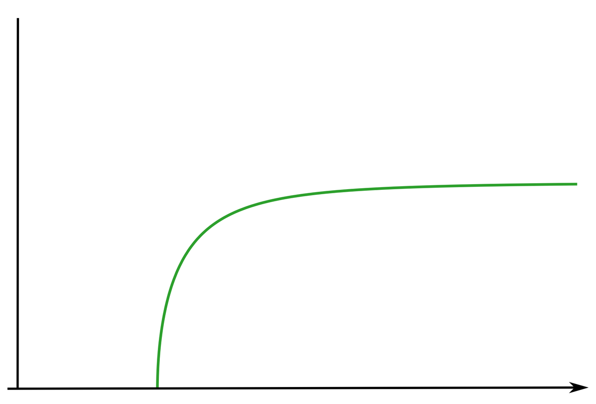
For 0 < 1, the potential cannot trap N fermions.

For 1, as the potential energy has to be finite $h\hat{V}i = \sum_k \int_1^1 dxj\ _k(x)j^2 V(x) < 1$, $_k(0) = 0$.

The behaviour at the edge depends on the value of : [2]

For 1 < 2, same behaviour as for the hard box
! The kernel close to the hard edge in $x = 0$ is K_1^e

For = 2, critical case, same behaviour as LUE with = + 1=2
! The kernel close to the hard edge in $x = 0$ is K_J



For > 2, a gap opens between $x = 0$ and the edge of the density
! The kernel at the edge of the density in $r_e > 0$ is K_1^{soft}

Fermions and general JUE

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References

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[3] F. D. Cunden, F. Mezzadri, N. O' Connell, J. Stat. Phys. **171** (5), 768-801 (2018).

[4] B. Lacroix-A-Chez-Toine, S. N. Majumdar, G. Schehr, arXiv preprint: **1809.05835** (2018).