A Transition in Gap Probabilities - from 1 Gap to 2

- Let K_s be the operator on $L^2(A)$ with the kernel

$$K_s(x;y) = \frac{\sin s(x \quad y)}{(x \quad y)}:$$

- For a wide class of random matrices , the probability of finding no eigenvalues of a random matrix M on a set $sA/$ in the bulk scaling limit is given by the Fredholm determinant $\det(I \quad K_s)_A$.
- We focus on the asymptotic behaviour of $\det(I \quad K_s)_A$ as $s \ ! \ 1$, in particular the probability of large gaps $sA/$ where A is composed of one or two intervals.

- We have seen results for 1 interval and 2 intervals.
- The formula for 2 intervals held on

$$A = (\; ; \quad) \bigcup (\; ; \;);$$

for fixed $\; ; \; ;$ as $s \to \infty$.

- We study the case where $\; \to 0$ simultaneously as $s \to \infty$. We present results for the regime $s \; \to 0$.

Theorem (F, Krasovsky)

For the constant $= \frac{4}{1 \quad 1}$, write in the form

$$= \quad e^{\frac{s\sqrt{j \quad j}}{w}}; \quad w = k + x; \quad k \in \mathbb{N}; \; x \in [\;1/2; 1/2).$$

Then, as $s \to \infty$ uniformly for $\in (0; \;_0)$, where $s \;_0 \to 0$,

$$\log\det(I \quad K_s)_A = \log\det(I \quad K_s)_{(\; ; \;)}$$
$$+ s\sqrt{j \quad j} \quad w \quad \frac{x^2}{w} \quad + c(k) + \quad (x) + O(\max\{s \;_0; s \;^{1}\});$$

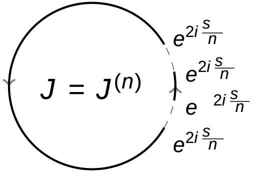
Let G be the Barnes' G-function ($G(k+1) = \;(k)G(k)$ and $G(1) = 1$). Then

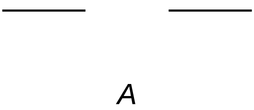
$$c(k) = \log\left(\frac{2^{2k^2 \quad k}}{k}\frac{G(k+1)^4}{G(2k+1)}\right):$$

- Alternatively, let j

- This transition from one gap to two gaps has an interesting parallel in the unitary random matrix ensembles exhibiting a transition from one-cut support to two-cut support as in the "birth of a cut", where asymptotic results for the correlation kernel where given simultaneously by Bertola & Lee, Claeys, Mo ('07)–('09). Fluctuations of the same type were also witnessed here.

Method of proof





- Define the Toeplitz determinant on the interval J by

$$D_n(\ \ ) = \det(f_{j\ \ k})_{j;k=1}^{n}; \quad f_j = \int_J e^{\ \ ij\ \ } \frac{d\ \ }{2\ \ }$$

- We have the following link between the Toeplitz determinant and the Fredholm determinant

$$\lim_{n \to \infty} D_{n;s}(\ \ ) = \det(I \quad K_s)_A.$$