Applications of Random Matrix Theory on Fiber Optical Communications

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Introduction

- Need for high speed infrastructure

Bandwidth (BW) hungry technologies emerging (~2 dB increase per year 1

Introduction

- <u>What is Optical MIMO</u>?

  Just like in wireless domain, in optical we can use N parallel transmission paths to greatly enhance the capacity of the system.

  – First paper on Optical MIMO: H. R. Stuart, “Dispersive multiplexing in multimode optical fiber,” Science 289(5477), 281–283 (2000)

  – Multi-Mode Fibers (MMF): Use multiple modes to carry information

  – Multi-Core Fibers (MCF): Utilize different optical paths of different cores in the same fiber (within the same cladding)

Introduction

Problem: Crosstalk phenomenon arises

Crosstalking between adjacent cores in MCF

Light beam scattering resulting in crosstalking in MMF

Due to :

- Extensive fiber length
- Bending of fiber
- Limited area with multiple power distributions
- Light beam scattering
- Non-linearities

Introduction

System Model

Consider a single-segment N- F K D Q Q H O O R V V O H V V W N_t ” R S W L F D O ¿ transmitting channels excited, N_r ” 1 receiving channels coherently. The $2N \times 2N$ scattering matrix is

$$S \quad \begin{pmatrix} r_t & t \\ t^T & r_r \end{pmatrix} \quad (S = S^T)$$

Only t (Haar-distributed: $t^\dagger t = tt^\dagger = I_N$) sub-matrix is of interest (r is ~0).

Generally N_r , N_t < N:

- Other channels may be used from different, parallel transceivers
- Modelling of loss: additional energy lost during propagation

System Model

Only t (Haar-distributed: $t^\dagger t = t t^\dagger = I_N$) sub-matrix is of interest (r is ~0).

$$t = \underbrace{\begin{bmatrix} t_{11} & \cdots & t_{1N} \\ \vdots & & \vdots \\ t_{N1} & \cdots & t_{NN} \end{bmatrix}}_{}$$

(the first N_t columns and first N_r rows are marked as N_t and N_r)

Define $N_t \times N_r$ matrix U as $\quad U = P_{N_t}^T S P_{N_r}$

- where P projection operator:

$$P_{N_t} = \begin{bmatrix} I_{N_t} \\ 0 \end{bmatrix} \begin{matrix} N_t \\ N-N_t \end{matrix}$$

Information Metrics

– Outage Probability:

$$P_{out}(r) \le \text{Prob}\{ I_N \le N_t r \} = E_U\{ 1_e(N_t r > I_N(U)) \}$$

- Optimal: Assumes infinite codewords

What is the price of finite codelengths?

– Gallager error bound for M-length code:

$$P_{err}(r) \le E_U\left[\exp\left[-M \max_{0 \le k \le 1} \left(-N_t k r + k \log\det\left[I + \frac{\rho}{1+k} U^{\dagger} U \right] \right) \right] \right]$$

Coulomb– Gas Analogy

- Joint probability distribution of eigenvalues of U
  - –

Generalized MP equation

- Use Tricomitheorem to calculate p(x)

- Obtain closed form expr. for energy S[p]

- E.g. ! ; n>0

$$p(x) = \frac{\sqrt{(b-x)(x-a)}}{2\pi x(1-\lambda)} \left(\frac{n(1-\lambda)}{(1-x)\sqrt{(1-a)(1-b)}} - \frac{\beta - 1}{x\sqrt{ab}} \right)$$

  - a, b, k calculated from

$$p(b) = p(a) = 0$$

$$r = \int_a^b dx\, p(x) \log(1-\lambda x)$$

$$1 = \int_a^b dx\, p(x)$$

Generalized MP equation

- In general

	S_{0b}	S_{ab}	S_{01}	S_{a1}
	a=0 b<1	a>0 b<1	a=0 b=1	a>0 b=1
n=0;	$r<r_{c1}$	--	$r_{c1} < r < r_{c2}$	$r>r_{c2}$
n>0;	$r<r_{c3}$	$r>r_{c3}$	--	--
n=0; >1	--	$r<r_{c4}$	--	$r>r_{c4}$
n>0; >1	--	all r	--	--

- Phase transitions (a=0 to a>0) etc are third order
  - discontinuous $S'''(r_c)$
  - Relation to Tracy-Widom (?)

Distribution Density of r

Finally

$$f(r) \mid N_t \frac{e^{N_t^2 \; S(r) \quad S(r_{erg})}}{\sqrt{2 \pi v_{erg}}}$$

– where

$$v_{erg} \quad \frac{1}{S''(r_{erg})} \quad \log \left(\frac{\sqrt{1 \quad b/b_0} \quad \sqrt{1 \quad a/a_0}^{\;2}}{4\sqrt{1 \quad a/a_0}\sqrt{1 \quad b/b_0}} \right)$$

is the variance at the peak of the distribution.

Numerical Simulations (! and $n_0 > 0$)

The LD approach demonstrates better behavior, following Monte Carlo.

Numerical Simulations (! and $n_0 > 0$)

The LD approach demonstrates better behavior, following Monte Carlo.

For small values of N_r, N_t and N_0, the discrepancy is minimal

Finite Block-Length Error Probability

- Here we need

  - where

-

Numerical Simulations (!   and $n_0 = 0$)

More Realistic Channel

- Chaotic cavity picture:

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- Mutual Information metric:

$$I(G) \quad \log\det I \quad \Theta_r \quad H_0 \quad G \quad i + {}^{1}P_t P_t \quad H_0 \quad G \quad i + {}^{1}P_r$$

- I – Distribution (mean-variance) can be obtained using replica theory

$$E\, I(G) \;@\; I_1 \quad I_2$$

$$I_1 \quad \log\det(U \quad G \quad t)(1 \quad r) \quad H_0 \quad p)^2 \;@\; N\,\mathrm{tr} \quad p^2$$

$$r \quad \frac{1}{N}\mathrm{Tr}\left[\frac{(1 \quad r)}{(U \quad G \quad t)(1 \quad r) \quad H_0 \quad p)^2}\right]$$

$$p \quad \frac{1}{N}\mathrm{Tr}\left[\frac{(H_0 \quad p)}{(U \quad G \quad t)(1 \quad r) \quad H_0 \quad p)^2}\right]$$

$$t \quad \frac{1}{N}\mathrm{Tr}\left[\frac{(U \quad G \quad t)}{(U \quad G \quad t)(1 \quad r) \quad H_0 \quad p)^2}\right]$$

– I_2 same with! – also expressions for variance

Numerical Simulations